A Reification of a Strategy for Geometry Theorem Proving

Noboru Matsuda[1] and Kurt VanLehn[2] *

[1] Intelligent Systems Program, University of Pittsburgh,
[2] Learning Research and Development Center, University of Pittsburgh
3939 O'Hara Street, Pittsburgh PA 15260

This study addresses a novel technique to build a graphical user interface (GUI) for an intelligent tutoring system (ITS) to help students to learn geometry theorem proving with construction – one of the most challenging and creative parts of elementary geometry. Students' task is not only to prove theorems, but also to construct missing points and/or segments to complete a proof (called *auxiliary lines*). The problem space of theorem proving with construction is generally huge, thus understanding a search strategy is a key issue for students to succeed in this domain. Two major challenges in building a GUI for an intelligent learning environment are (a) to build a theorem prover that is capable of construction, and (b) to establish a cognitive model of understanding a complex problem-solving strategy.

So far, we have built a geometry theorem prover, GRAMY, which can automatically construct auxiliary lines when needed to complete a proof. GRAMY utilizes a simple single heuristic for construction, which says that "apply a known axiom or theorem backwards by overlapping the related configuration with the diagram given in the problem while allowing the match to omit segment(s) in the configuration." The auxiliary lines are those which match the omitted segments. Surprisingly, this simple heuristic works very well. This suggests that it might be possible to teach students how to construct auxiliary lines.

In order to develop an ITS based on GRAMY, we need a GUI to display the reasoning in a graphical, manipulable form — to "reify" (make real) the reasoning. Some common techniques for reification have flaws, so suggest a new one. Our basic idea is to reify the search process rather than the structure of the ultimate solution. The resulting GUI shows an intermediate state of proof. It consists of the diagram of a theorem to prove, and applicable axioms (or theorems) in that state. Using tools that look like those of a web browser, the student can select an applicable axiom to proceed a state, or go back and forth between states.

We show how this reification technique can be applied to teach students three basic search strategies; forward chaining, backward chaining, and backing up at dead-ends. We discuss why this reification model should be better than the ones that reifies the entire solution space as trees.

---

* Email: `mazda@isp.pitt.edu` and `vanlehn@cs.pitt.edu` This research was supported by NSF grant number 9720359 to CIRCLE: Center for Interdisciplinary Research on Constructive Learning Environments. `http://www.pitt.edu/~circle`