APPENDIX A: An Example of Scaffolding of the Forward AGT

Participant FC020 working on the problem #7

0021.0 T "Let's make an assertion by applying a postulate to the known facts."
0046.9 S :CLICK-OK
0047.6 T "This is done by 2 steps, which are [Assert a new proposition] and [Justify Proposition]."
0048.8 S :CLICK-OK
0049.4 T "It's time to assert a new proposition that can be logically concluded either from the proposition already in the Proof Table or the problem figure itself."
0050.1 S :CLICK-OK
0050.6 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0083.0 S (:STRING (:EQ (:S B D) (:S B D)))
0083.5 T "Well, BD = BD is not a proposition that I want you to consider now."
0085.9 S :CLICK-OK
0086.5 T "Please try again."
0088.1 S :CLICK-OK
0088.7 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0109.8 S (:STRING (:EQ (:A D M Q) (:A P M B)))
0110.1 T "Good. ∠DMQ = ∠PMB is an appropriate proposition to assert now. Keep going."
0112.3 S :CLICK-OK
0113.9 T "Next step is to justify vertical angles"
0114.7 S :CLICK-OK
0115.3 T "Now it's your turn. Please enter a justification of ∠BMP = ∠DMQ into the highlighted cell in the Proof Table."
0121.1 S (:STRING VERANG)
0121.5 T "Good. Keep going."
0123.4 S :CLICK-OK
0125.0 T "It seems we need some construction to complete a proof"
0125.7 S :CLICK-OK
0126.3 T "This is done by 2 steps, which are [Pick a postulate for construction] and [Do construction]."
0126.7 S :CLICK-OK
0127.6 T "Select a postulate that helps you complete the proof."
0128.0 S :CLICK-OK
0128.6 T "A construction is done by finding a postulate that partially matches with the problem figure. The idea is that once a construction is properly done, then the postualte would apply and you can assert a new proposition into the Proof Table."
0128.9 S :CLICK-OK
0129.5 T "Now, enter a postulate name, in the text box below, that you think would be useful for the proof."
0154.0 S (:STRING SAS)

0154.6 T "Well, good guess. But, SAS is not an appropriate postulate in this situation. I'd recommend you to use CPCTC instead."
0158.0 S :CLICK-OK
0158.6 T "So, keep it in your mind, and think about how you can make a construction with CPCTC."
0162.0 S :CLICK-OK
0162.9 T "Look at the Postulate Browser window, it now shows how the postulate CPCTC would help to make a construction. You need to draw segments that are not in the problem figure to make overlapping shown in the Postulate Browser window."
0164.9 S :CLICK-OK
0165.5 T "Make a construction based on the overlapping"
0168.9 S :CLICK-OK
0169.8 T "Draw new segments by connecting two points. To connect two points, click -- no dragging, no double clicking -- on the first point, move the mouse, and then click on the second point."
0187.5 S (:CONNECT-POINTS (:S D P)))
0187.8 T "Hum, good try, but that sounds not right for me. Why don't you draw a line by connecting B and P instead. That way, you can make a parfect overlap for the postulte you have been selected. Remember, a construction is done so that a desired postulate would have a parfect match with the problem figure."
0197.4 S :CLICK-OK
0198.0 T "Draw new segments by connecting two points. To connect two points, click -- no dragging, no double clicking -- on the first point, move the mouse, and then click on the second point."
0203.5 S (:CONNECT-POINTS (:S B P)))
0203.8 T "You are right on the target. Click [OK] to draw more lines."
0206.5 S :CLICK-OK
0207.1 T "Draw new segments by connecting two points. To connect two points, click -- no dragging, no double clicking -- on the first point, move the mouse, and then click on the second point."
0216.4 S (:CONNECT-POINTS (:S D Q)))
0216.7 T "You are right on the target. Click [OK] to proceed."
0217.8 S :CLICK-OK
0218.4 T "Okay, that completes the construction we need."
0218.9 S :CLICK-OK
0219.5 T "Okay, we have completed a necessary construction. Go ahead to the next proof step."
0220.7 S :CLICK-OK
0221.6 T "Let's make an assertion by applying a postulate to the known facts."
0222.4 S :CLICK-OK
0223.0 T "This is done by 2 steps, which are [Assert a new proposition] and [Justify Proposition]."
0223.4 S :CLICK-OK

0224.1 T "It's time to assert a new proposition that can be logically concluded either from the proposition already in the Proof Table or the problem figure itself."
0224.4 S :CLICK-OK
0225.0 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0248.2 S (:STRING (:CONG (:TRIANGLE (:P A) (:P P) (:P B) (:TRIANGLE (:P C) (:P Q) (:P D))))
0248.6 T "ΔAPB ≡ ΔCQD is actually a part of the proof. However, it's too soon to assert it. You need to assert some other propositions before ΔAPB ≡ ΔCQD"
0258.8 S :CLICK-OK
0259.4 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0271.0 S (:STRING (:EQ (:S B P) (:S Q D)))
0271.4 T "BP = QD is actually a part of the proof. However, it's too soon to assert it. You need to assert some other propositions before BP = QD"
0274.3 S :CLICK-OK
0274.9 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0348.8 S (:STRING (:EQ (:A B M Q) (:A P M D)))
0349.3 T "Oops, wrong again. Please enter a proposition that must be a part of the proof and that can be concluded only from the propositions that are already in the Proof Table. You can not skip a proof step neither."
0352.7 S :CLICK-OK
0440.0 S (:STRING (:CONG (:TRIANGLE (:P B) (:P A) (:P P) (:TRIANGLE (:P D) (:P Q) (:P C))))
0440.4 T "Well, you need to enter ΔBMP ≡ ΔDMQ in the highlighted cell."
0451.9 S :CLICK-OK
0452.6 T "Please input a right proposition, which is ΔBMP ≡ ΔDMQ, in the Proof Table."
0454.7 S :CLICK-OK
0467.2 S (:STRING (:CONG (:TRIANGLE (:P B) (:P M) (:P P) (:TRIANGLE (:P D) (:P M) (:P Q))))
0467.6 T "Good."
0468.8 S :CLICK-OK
0471.3 T "Now, we need to find and apply a postulate to justify ΔBMP ≡ ΔDMQ that has been just asserted."
0471.8 S :CLICK-OK
0472.4 T "This is done by 2 steps, which are [Select a postulate] and [Execute the postulate]."
0472.9 S :CLICK-OK
0473.7 T "Next step is to select a postulate that can be a justification of ΔBMP ≡ ΔDMQ"

0474.0 S :CLICK-OK
0474.6 T "This is done by 3 steps, which are [Pick a postulate], [Overlap configuration], and [Transform conditional]."
0475.1 S :CLICK-OK
0475.8 T "The justification cell in the Proof Table is about to have a postulate's name."
0476.2 S :CLICK-OK
0476.8 T "Now, it's you turn to pick a postulate. Enter a postulate name into the highlighted justification cell."
0498.1 S (:STRING CPCTC)
0498.6 T "You could try to apply CPCTC, but there is a better one. Try again. Enter a postulate name that can justify ∆BMP ≡ ∆DMQ."
0500.6 S :CLICK-OK
0511.7 S (:STRING SAS)
0512.0 T "Good. Go on to the next step."
0514.0 S :CLICK-OK
0514.8 T "To apply SAS, overlap it onto the problem figure."
0515.2 S :CLICK-OK
0516.1 T "There is nothing to do with the Proof Table for this step. But, remember you need to overlap the configuration of the postulate SAS onto the problem figure. Please do so in your mind. When finished, click [OK] button to go on the next step."
0516.4 S :CLICK-OK
0517.0 T "Now, it's time to recall the premises and a consequence of SAS."
0517.4 S :CLICK-OK
0518.4 T "There is nothing to enter into the Proof Table. But, remember you need to identify the premises and the consequence of the postulate SAS. Please do it now in your mind. When finished, click [OK] to go on to the next step."
0518.7 S :CLICK-OK
0519.3 T "This is done by 2 steps, which are [Instantiate premises] and [Assert line numbers]."
0520.0 S :CLICK-OK
0520.6 T "Now, let's see what we exactly need to justify ∆BMP ≡ ∆DMQ. Namely, state the premise with the labels appearing in the problem figure."
0521.0 S :CLICK-OK
0521.6 T "There is nothing to enter into the Proof Table for this step. But, remember you need to describe the postulate's premises XY = UV, YZ = VW, and ∆XYZ ≡ ∆UVW with the labels used in the problem figure. Please do so in your mind. When finished, click [OK] to proceed to the next step."
0522.8 S :CLICK-OK
0523.5 T "The next step is to enter the line numbers of the premises."
0524.5 S :CLICK-OK
0525.1 T "It's your turn to do this step."

0531.0 S (:STRING 5)
0557.1 S (:STRING 3)
0563.2 S (:STRING 2)
0563.6 T "Gook job!! You have entered all the premise numbers. Click [OK] to proceed to the next step."
0565.1 S :CLICK-OK
0565.7 T "We've completed enough to justify proposition ∆BMP ≡ ∆DMQ. Go on to the next proof step."
0566.0 S :CLICK-OK
0567.5 T "Let's make an assertion by applying a postulate to the known facts."
0568.0 S :CLICK-OK
0568.6 T "This is done by 2 steps, which are [Assert a new proposition] and [Justify Proposition]."
0569.0 S :CLICK-OK
0569.7 T "It's time to assert a new proposition that can be logically concluded either from the proposition already in the Proof Table or the problem figure itself."
0570.0 S :CLICK-OK
0570.6 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0587.7 S (:STRING (:EQ (:S P B) (:S D Q)))
0588.0 T "Good. PB = DQ is an appropriate proposition to assert now. Keep going."
0589.2 S :CLICK-OK
0591.8 T "Now, we need to find and apply a postulate to justify BP = DQ that has been just asserted."
0592.2 S :CLICK-OK
0592.8 T "This is done by 2 steps, which are [Select a postulate] and [Execute the postulate]."
0593.1 S :CLICK-OK
0593.7 T "Next step is to select a postulate that can be a justification of BP = DQ"
0594.2 S :CLICK-OK
0594.9 T "This is done by 3 steps, which are [Pick a postulate], [Overlap configuration], and [Transform conditional]."
0595.9 S :CLICK-OK
0596.6 T "The justification cell in the Proof Table is about to have a postulate's name."
0597.0 S :CLICK-OK
0597.7 T "Now, it's you turn to pick a postulate. Enter a postulate name into the highlighted justification cell."
0603.6 S (:STRING CPCTC)
0603.9 T "Good. Go on to the next step."
0605.3 S :CLICK-OK
0606.1 T "To apply CPCTC, overlap it onto the problem figure."

0606.6 S :CLICK-OK

0607.5 T "There is nothing to do with the Proof Table for this step. But, remember you need to overlap the configuration of the postulate CPCTC onto the problem figure. Please do so in your mind. When finished, click [OK] button to go on the next step."

0607.8 S :CLICK-OK

0608.4 T "Now, it's time to recall the premises and a consequence of CPCTC."

0608.8 S :CLICK-OK

0609.6 T "There is nothing to enter into the Proof Table. But, remember you need to identify the premises and the consequence of the postulate CPCTC. Please do it now in your mind. When finished, click [OK] to go on to the next step."

0609.9 S :CLICK-OK

0610.5 T "This is done by 2 steps, which are [Instantiate premises] and [Assert line numbers]."

0611.1 S :CLICK-OK

0611.8 T "Now, let's see what we exactly need to justify BP = DQ. Namely, state the premise with the labels appearing in the problem figure."

0612.3 S :CLICK-OK

0612.9 T "There is nothing to enter into the Proof Table for this step. But, remember you need to describe the postulate's premise $\Delta XYZ \equiv \Delta UVW$ with the labels used in the problem figure. Please do so in your mind. When finished, click [OK] to proceed to the next step."

0613.5 S :CLICK-OK

0614.2 T "The next step is to enter the line numbers of the premises."

0615.7 S :CLICK-OK

0616.3 T "It's your turn to do this step."

0709.0 S (:STRING 6)

0709.3 T "Gook job!! You have entered all the premise numbers. Click [OK] to proceed to the next step."

0711.1 S :CLICK-OK

0711.7 T "We've completed enough to justify proposition BP = DQ. Go on to the next proof step."

0712.0 S :CLICK-OK

0713.4 T "Let's make an assertion by applying a postulate to the known facts."

0714.0 S :CLICK-OK

0714.6 T "This is done by 2 steps, which are [Assert a new proposition] and [Justify Proposition]."

0714.9 S :CLICK-OK

0715.5 T "It's time to assert a new proposition that can be logically concluded either from the proposition already in the Proof Table or the problem figure itself."

0715.9 S :CLICK-OK
0716.5 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0727.8 S (:STRING (:EQ (:A B A P) (:A D C Q)))
0728.2 T "∆BAP ≡ ∆DCQ is actually a part of the proof. However, it's too soon to assert it. You need to assert some other propositions before ∆BAP ≡ ∆DCQ"
0731.3 S :CLICK-OK
0732.0 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0749.2 S (:STRING (:CONG (:TRIANGLE (:P B) (:P A) (:P P) (:TRIANGLE (:P D) (:P C) (:P Q))))
0749.5 T "Good. ∆BAP ≡ ∆DCQ is an appropriate proposition to assert now. Keep going."
0751.2 S :CLICK-OK
0753.9 T "Now, we need to find and apply a postulate to justify ∆BAP ≡ ∆DCQ that has been just asserted."
0754.4 S :CLICK-OK
0755.0 T "This is done by 2 steps, which are [Select a postulate] and [Execute the postulate]."
0755.3 S :CLICK-OK
0755.9 T "Next step is to select a postulate that can be a justification of ∆BAP ≡ ∆DCQ"
0757.3 S :CLICK-OK
0757.9 T "This is done by 3 steps, which are [Pick a postulate], [Overlap configuration], and [Transform conditional]."
0758.4 S :CLICK-OK
0759.1 T "The justification cell in the Proof Table is about to have a postulate's name."
0759.7 S :CLICK-OK
0760.3 T "Now, it's you turn to pick a postulate. Enter a postulate name into the highlighted justification cell."
0770.3 S (:STRING CPCTC)
0770.7 T "You could try to apply CPCTC, but there is a better one. Try again. Enter a postulate name that can justify ∆BAP ≡ ∆DCQ."
0772.2 S :CLICK-OK
0778.9 S (:STRING SSS)
0779.2 T "Good. Go on to the next step."
0781.0 S :CLICK-OK
0781.7 T "To apply SSS, overlap it onto the problem figure."
0782.3 S :CLICK-OK
0783.2 T "There is nothing to do with the Proof Table for this step. But, remember you need to overlap the configuration of the postulate SSS onto the problem figure. Please do so in your mind. When finished, click [OK] button to go on the next step."
0783.4 S :CLICK-OK

0784.0 T "Now, it's time to recall the premises and a consequence of SSS."
0784.5 S :CLICK-OK
0785.4 T "There is nothing to enter into the Proof Table. But, remember you need to identify the premises and the consequence of the postulate SSS. Please do it now in your mind. When finished, click [OK] to go on to the next step."
0785.7 S :CLICK-OK
0786.3 T "This is done by 2 steps, which are [Instantiate premises] and [Assert line numbers]."
0786.9 S :CLICK-OK
0787.6 T "Now, let's see what we exactly need to justify ∆BAP ≡ ∆DCQ. Namely, state the premise with the labels appearing in the problem figure."
0787.9 S :CLICK-OK
0788.5 T "There is nothing to enter into the Proof Table for this step. But, remember you need to describe the postulate's premises YZ = VW, XY = UV, and XZ = UW with the labels used in the problem figure. Please do so in your mind. When finished, click [OK] to proceed to the next step."
0791.5 S :CLICK-OK
0792.1 T "The next step is to enter the line numbers of the premises."
0793.8 S :CLICK-OK
0794.4 T "It's your turn to do this step."
0799.2 S (:STRING 7)
0803.4 S (:STRING 4)
0806.9 S (:STRING 1)
0807.3 T "Gook job!! You have entered all the premise numbers. Click [OK] to proceed to the next step."
0808.7 S :CLICK-OK
0809.3 T "We've completed enough to justify proposition ∆BAP ≡ ∆DCQ. Go on to the next proof step."
0809.6 S :CLICK-OK
0811.1 T "Let's make an assertion by applying a postulate to the known facts."
0811.5 S :CLICK-OK
0812.1 T "This is done by 2 steps, which are [Assert a new proposition] and [Justify Proposition]."
0812.4 S :CLICK-OK
0813.0 T "It's time to assert a new proposition that can be logically concluded either from the proposition already in the Proof Table or the problem figure itself."
0813.3 S :CLICK-OK
0813.9 T "Now, it's your turn. Enter a proposition that can be justified with the propositions already in the Proof Table."
0819.7 S (:STRING (:EQ (:A B A P) (:A D C Q)))

0820.0 T "Good. ∆BAP ≡ ∆DCQ is an appropriate proposition to assert now. Keep going."
0821.6 S :CLICK-OK
0824.4 T "Now, we need to find and apply a postulate to justify ∆BAP ≡ ∆DCQ that has been just asserted."
0826.0 S :CLICK-OK
0826.6 T "This is done by 2 steps, which are [Select a postulate] and [Execute the postulate]."
0826.9 S :CLICK-OK
0827.6 T "Next step is to select a postulate that can be a justification of ∆BAP ≡ ∆DCQ"
0828.5 S :CLICK-OK
0829.1 T "This is done by 3 steps, which are [Pick a postulate], [Overlap configuration], and [Transform conditional]."
0829.4 S :CLICK-OK
0830.1 T "The justification cell in the Proof Table is about to have a postulate's name."
0830.4 S :CLICK-OK
0831.0 T "Now, it's you turn to pick a postulate. Enter a postulate name into the highlighted justification cell."
0836.9 S (:STRING CPCTC)
0837.3 T "Good. Go on to the next step."
0839.4 S :CLICK-OK
0840.1 T "To apply CPCTC, overlap it onto the problem figure."
0840.8 S :CLICK-OK
0841.7 T "There is nothing to do with the Proof Table for this step. But, remember you need to overlap the configuration of the postulate CPCTC onto the problem figure. Please do so in your mind. When finished, click [OK] button to go on the next step."
0842.0 S :CLICK-OK
0842.6 T "Now, it's time to recall the premises and a consequence of CPCTC."
0843.2 S :CLICK-OK
0844.1 T "There is nothing to enter into the Proof Table. But, remember you need to identify the premises and the consequence of the postulate CPCTC. Please do it now in your mind. When finished, click [OK] to go on to the next step."
0844.4 S :CLICK-OK
0845.0 T "This is done by 2 steps, which are [Instantiate premises] and [Assert line numbers]."
0845.5 S :CLICK-OK
0846.2 T "Now, let's see what we exactly need to justify ∆BAP ≡ ∆DCQ. Namely, state the premise with the labels appearing in the problem figure."
0846.5 S :CLICK-OK

0847.1 T "There is nothing to enter into the Proof Table for this step. But, remember you need to describe the postulate's premise ΔXYZ ≡ ΔUVW with the labels used in the problem figure. Please do so in your mind. When finished, click [OK] to proceed to the next step."
0847.4 S :CLICK-OK
0848.1 T "The next step is to enter the line numbers of the premises."
0848.4 S :CLICK-OK
0849.0 T "It's your turn to do this step."
0852.9 S (:STRING 8)
0853.2 T "Gook job!! You have entered all the premise numbers. Click [OK] to proceed to the next step."
0854.6 S :CLICK-OK
0855.2 T "We've completed enough to justify proposition ΔBAP ≡ ΔDCQ. Go on to the next proof step."
0855.5 S :CLICK-OK
0856.5 T "Good job!!!"
0857.0 S :CLICK-OK
0857.6 T "We've complete the proof. Now, go ahead to the next problem.

APPENDIX B: An Example of Scaffolding of Backward AGT

Participant BC027 working on the problem #10

0645.4 T "Let's make one step toward a complete proof. Namely, justify an unjustified proposition in the Proof Table."
0927.1 S :CLICK-OK
0927.7 T "This is done by 2 steps, which are [Select proposition to justify] and [Apply a postulate]."
0928.3 S :CLICK-OK
0928.9 T "The first step is to specify an unjustified proposition in the Proof Table."
0929.4 S :CLICK-OK
0930.0 T "It's your turn to do this step."
0931.1 S (:MOUSE-CLICK PROOFTABLE (:EQ (:S A H) (:S D H)))
0931.5 T "Good. Go on to the next step."
0932.4 S :CLICK-OK
0933.2 T "Next step is to select a postulate and apply it to justify the proposition AH = DH."
0937.9 S :CLICK-OK
0938.5 T "This is done by 2 steps, which are [Select a postulate] and [Execute the postulate]."
0939.2 S :CLICK-OK
0939.9 T "Next step is to select a postulate that can be a justification of AH = DH"
0940.3 S :CLICK-OK
0940.9 T "This is done by 4 steps, which are [Pick a postulate], [Overlap configuration], [Do construction], and [Transform conditional]."
0941.5 S :CLICK-OK
0942.3 T "The justification cell in the Proof Table is about to have a postulate's name."
0943.0 S :CLICK-OK
0943.6 T "Please enter a postulate's name into the highlighted cell in the Proof Table. You are welcome to use the Postulate Browser to remember each postulate."
1045.0 S (:STRING TRIM)
1045.4 T "Well, you made a good guess, but unfortunately, TRIM won't justify AH = DH in this case. Why don't you apply MTRI instead."
1049.3 S :CLICK-OK
1050.3 T "To apply MTRI, overlap it onto the problem figure."
1055.5 S :CLICK-OK
1056.4 T "There is nothing to do with the Proof Table for this step. But, remember you need to overlap the configuration of the postulate MTRI onto the problem figure. Please do so in your mind. When finished, click [OK] button to go on the next step."
1191.5 S :CLICK-OK
1192.1 T "As you see, the postulate MTRI can not be overlapped onto the problem figure since some segments are missing. Draw segments

so that the postulate have a perfect match. You may need to draw more than one line to make a perfect match."

1193.8 S :CLICK-OK

1194.4 T "Draw new segments by connecting two points. To connect two points, click -- no dragging, no double clicking -- on the first point, move the mouse, and then click on the second point."

1203.0 S (:CONNECT-POINTS (:S F H))

1203.4 T "Hum, good try, but that sounds not right for me. Why don't you draw a line by connecting B and D instead. That way, you can make a parfect overlap for the postulte you have been selected. Remember, a construction is done so that a desired postulate would have a parfect match with the problem figure."

1214.0 S :CLICK-OK

1214.7 T "Draw new segments by connecting two points. To connect two points, click -- no dragging, no double clicking -- on the first point, move the mouse, and then click on the second point."

1220.7 S (:CONNECT-POINTS (:S B D))

1221.0 T "You are right on the target. Click [OK] to proceed."

1222.1 S :CLICK-OK

1222.7 T "Okay, that completes the construction we need."

1224.0 S :CLICK-OK

1224.6 T "Now, it's time to recall the premises and a consequence of MTRI."

1226.3 S :CLICK-OK

1227.2 T "There is nothing to enter into the Proof Table. But, remember you need to identify the premises and the consequence of the postulate MTRI. Please do it now in your mind. When finished, click [OK] to go on to the next step."

1238.6 S :CLICK-OK

1239.2 T "Now, it's time to apply the selected postulate backwards on AH = DH, namely identify premises of the postulate and put them onto the proof table as unjustified propositions."

1239.5 S :CLICK-OK

1240.2 T "This is done by 4 steps, which are [Instantiate premises], [Check duplication], [Assert premises], and [Assert line numbers]."

1240.6 S :CLICK-OK

1241.6 T "Now, let's see what we exactly need to justify AH = DH. Namely, state the premise with the labels appearing in the problem figure."

1241.9 S :CLICK-OK

1242.5 T "There is nothing to enter into the Proof Table for this step. But, remember you need to describe the postulate's premises WX = WZ and VW // YZ with the labels used in the problem figure. Please do so in your mind. When finished, click [OK] to proceed to the next step."

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1242.8   S    :CLICK-OK
1243.4   T    "For each of the premises, check if it has been asserted in the
              proof table or not."
1243.8   S    :CLICK-OK
1244.4   T    "Again, there is nothing you need to enter into the Proof Table
              here.  You need to check if the premises of the justification
              you are about to make are already in the Proof Table.  Please do
              so in your mind, and click [OK] when finished."
1244.7   S    :CLICK-OK
1245.3   T    "Now, we need to assert the premises that are not yet in the
              proof table."
1245.6   S    :CLICK-OK
1281.0   S    (:STRING (:EQ (:S A E) (:S E B)))
1281.3   T    "Good.  AE = EB must be asserted in the Proof Table.  Enter more
              premises."
1325.3   S    (:STRING (:PARALLEL (:S E H) (:S B D)))
1325.6   T    "Good.  EH // BD must be asserted in the Proof Table.  Enter
              more premises."
1326.2   T    "The next step is to enter the line numbers of the premises."
1328.3   S    :CLICK-OK
1328.9   T    "It's your turn to do this step."
1335.9   S    (:STRING 2)
1337.8   S    (:STRING 3)
1338.2   T    "Gook job!!  You have entered all the premise numbers.  Click
              [OK] to proceed to the next step."
1341.3   S    :CLICK-OK
1341.9   T    "We've completed enough to justify proposition AH = DH.  Go on
              to the next proof step."
1342.2   S    :CLICK-OK
1343.3   T    "Let's make one step toward a complete proof. Namely, justify an
              unjustified proposition in the Proof Table."
1344.5   S    :CLICK-OK
1345.2   T    "This is done by 2 steps, which are [Select proposition to
              justify] and [Justify given proposition]."
1345.8   S    :CLICK-OK
1346.4   T    "The first step is to specify an unjustified proposition in the
              Proof Table."
1347.0   S    :CLICK-OK
1347.7   T    "It's your turn to do this step."
1349.6   S    (:MOUSE-CLICK PROOFTABLE (:EQ (:S A E) (:S E B)))
1349.9   T    "Good.  Go on to the next step."
1352.2   S    :CLICK-OK
1353.1   T    "Justify a given proposition"
1353.5   S    :CLICK-OK
1354.1   T    "Now, it's your turn.  Please enter a justification of the
              postulate AE = BE into the highlighted cell."
```

1360.8 S (:STRING GIVEN)
1361.1 T "Good. Go on to the next step."
1362.8 S :CLICK-OK
1364.7 T "Let's make one step toward a complete proof. Namely, justify an unjustified proposition in the Proof Table."
1365.4 S :CLICK-OK
1366.0 T "This is done by 2 steps, which are [Select proposition to justify] and [Apply a postulate]."
1366.6 S :CLICK-OK
1367.2 T "The first step is to specify an unjustified proposition in the Proof Table."
1367.9 S :CLICK-OK
1368.6 T "It's your turn to do this step."
1369.9 S (:MOUSE-CLICK (:PARALLEL (:S E H) (:S B D)))
1370.2 T "Good. Go on to the next step."
1371.7 S :CLICK-OK
1372.7 T "Next step is to select a postulate and apply it to justify the proposition BD // EH."
1373.3 S :CLICK-OK
1374.0 T "This is done by 2 steps, which are [State transitive substitution] and [Execute the postulate]."
1374.4 S :CLICK-OK
1375.1 T "The target proposition BD // EH can be justified by the transitive-substitution postulate. There is no configuration associated in the Postulate Browser, but you can apply this postulate in the similar way as the other ones. Just no overlapping is required."
1375.9 S :CLICK-OK
1376.5 T "The proposition BD // EH that we are trying to justify can be justified by transitive substitution. So, I'd enter TRANS as its justification."
1387.3 S :CLICK-OK
1388.1 T "Now, it's time to apply the selected postulate backwards on BD // EH, namely identify premises of the postulate and put them onto the proof table as unjustified propositions."
1681.8 S :CLICK-OK
1682.4 T "This is done by 4 steps, which are [Instantiate premises], [Check duplication], [Assert premises], and [Assert line numbers]."
1684.6 S :CLICK-OK
1685.5 T "Now, let's see what exactly we need to justify BD // EH. Namely, state the premise with the labels appearing in the problem figure."
1686.2 S :CLICK-OK

1686.8 T "There is nothing you have to enter or click on the screen for this step. But, you need to mentally identify which two propositions are combined to conclude BD // EH."
1688.9 S :CLICK-OK
1689.5 T "In this case, BD // FG and FG // EH are the propositions that we need."
1736.9 S :CLICK-OK
1737.5 T "For each of the premises, check if it has been asserted in the proof table or not."
1739.3 S :CLICK-OK
1740.0 T "Again, there is nothing you need to enter into the Proof Table here. You need to check if the premises of the justification you are about to make are already in the Proof Table. Please do so in your mind, and click [OK] when finished."
1742.3 S :CLICK-OK
1742.9 T "Now, we need to assert the premises that are not yet in the proof table."
1744.1 S :CLICK-OK
1760.0 S (:STRING (:PARALLEL (:S B D) (:S E H)))
1760.5 T "You are mentioning point names that are not consistent with overlapping :TRANS onto the problem figure."
1767.6 S :CLICK-OK
1768.2 T "Please try again."
1768.8 S :CLICK-OK
1878.0 S (:STRING (:PARALLEL (:S B D) (:S F G)))
1878.3 T "Good. BD // FG must be asserted in the Proof Table. Enter more premises."
2000.7 S (:STRING-S PROPOSITION (:EQ (:S A E) (:S E B)))
2001.1 T "You are mentioning point names that are not consistent with overlapping :TRANS onto the problem figure."
2005.1 S :CLICK-OK
2005.7 T "Please try again."
2006.1 S :CLICK-OK
2065.0 S (:STRING (:PARALLEL (:S E H) (:S F G)))
2065.4 T "Good. EH // FG must be asserted in the Proof Table. Enter more premises."
2066.2 T "The next step is to enter the line numbers of the premises."
2068.7 S :CLICK-OK
2069.3 T "It's your turn to do this step."
2074.8 S (:STRING 4)
2075.8 S (:STRING 5)
2076.1 T "Gook job!! You have entered all the premise numbers. Click [OK] to proceed to the next step."
2077.5 S :CLICK-OK
2078.1 T "We've completed enough to justify proposition BD // EH. Go on to the next proof step."

2078.4 S :CLICK-OK
2080.5 T "Let's make one step toward a complete proof. Namely, justify an unjustified proposition in the Proof Table."
2083.0 S :CLICK-OK
2083.6 T "This is done by 2 steps, which are [Select proposition to justify] and [Apply a postulate]."
2083.9 S :CLICK-OK
2084.5 T "The first step is to specify an unjustified proposition in the Proof Table."
2084.9 S :CLICK-OK
2085.7 T "It's your turn to do this step."
2087.2 S (:MOUSE-CLICK (:PARALLEL (:S B D) (:S F G)))
2087.5 T "Good. Go on to the next step."
2096.4 S :CLICK-OK
2097.7 T "Next step is to select a postulate and apply it to justify the proposition BD // FG."
2109.8 S :CLICK-OK
2110.4 T "This is done by 2 steps, which are [Select a postulate] and [Execute the postulate]."
2111.9 S :CLICK-OK
2112.6 T "Next step is to select a postulate that can be a justification of BD // FG"
2112.9 S :CLICK-OK
2113.5 T "This is done by 3 steps, which are [Pick a postulate], [Overlap configuration], and [Transform conditional]."
2114.0 S :CLICK-OK
2114.7 T "The justification cell in the Proof Table is about to have a postulate's name."
2115.1 S :CLICK-OK
2115.7 T "Please enter a postulate's name into the highlighted cell in the Proof Table. You are welcome to use the Postulate Browser to remember each postulate."
2137.4 S (:STRING MTRI)
2137.8 T "Well, you made a good guess, but unfortunately, MTRI won't justify BD // FG in this case. Why don't you apply TRIM instead."
2151.3 S :CLICK-OK
2152.3 T "To apply TRIM, overlap it onto the problem figure."
2152.9 S :CLICK-OK
2153.9 T "There is nothing to do with the Proof Table for this step. But, remember you need to overlap the configuration of the postulate TRIM onto the problem figure. Please do so in your mind. When finished, click [OK] button to go on the next step."
2154.2 S :CLICK-OK
2154.8 T "Now, it's time to recall the premises and a consequence of TRIM."

2156.6 S :CLICK-OK
2157.5 T "There is nothing to enter into the Proof Table. But, remember you need to identify the premises and the consequence of the postulate TRIM. Please do it now in your mind. When finished, click [OK] to go on to the next step."
2157.8 S :CLICK-OK
2158.5 T "Now, it's time to apply the selected postulate backwards on BD // FG, namely identify premises of the postulate and put them onto the proof table as unjustified propositions."
2158.9 S :CLICK-OK
2159.6 T "This is done by 4 steps, which are [Instantiate premises], [Check duplication], [Assert premises], and [Assert line numbers]."
2160.5 S :CLICK-OK
2161.4 T "Now, let's see what we exactly need to justify BD // FG. Namely, state the premise with the labels appearing in the problem figure."
2161.8 S :CLICK-OK
2162.4 T "There is nothing to enter into the Proof Table for this step. But, remember you need to describe the postulate's premises WX = WZ and VX = VY with the labels used in the problem figure. Please do so in your mind. When finished, click [OK] to proceed to the next step."
2163.0 S :CLICK-OK
2163.6 T "For each of the premises, check if it has been asserted in the proof table or not."
2164.0 S :CLICK-OK
2164.6 T "Again, there is nothing you need to enter into the Proof Table here. You need to check if the premises of the justification you are about to make are already in the Proof Table. Please do so in your mind, and click [OK] when finished."
2165.3 S :CLICK-OK
2166.0 T "Now, we need to assert the premises that are not yet in the proof table."
2166.3 S :CLICK-OK
2216.5 S (:STRING (:EQ (:S B F) (:S F C)))
2216.9 T "Good. BF = FC must be asserted in the Proof Table. Enter more premises."
2256.7 S (:STRING (:EQ (:S D G) (:S G C)))
2257.1 T "Good. DG = GC must be asserted in the Proof Table. Enter more premises."
2258.1 T "The next step is to enter the line numbers of the premises."
2275.2 S :CLICK-OK
2275.9 T "It's your turn to do this step."
2283.0 S (:STRING 6)
2283.8 S (:STRING 7)

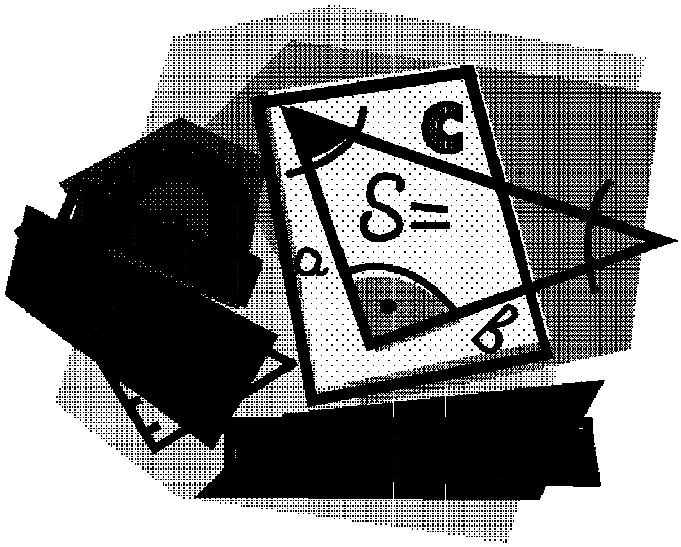
```
2284.1   T   "Gook job!!  You have entered all the premise numbers.  Click
             [OK] to proceed to the next step."
2286.5   S   :CLICK-OK
2287.1   T   "We've completed enough to justify proposition BD // FG.  Go on
             to the next proof step."
2287.4   S   :CLICK-OK
2288.6   T   "Let's make one step toward a complete proof. Namely, justify an
             unjustified proposition in the Proof Table."
2289.3   S   :CLICK-OK
2289.9   T   "This is done by 2 steps, which are [Select proposition to
             justify] and [Justify given proposition]."
2290.2   S   :CLICK-OK
2290.9   T   "The first step is to specify an unjustified proposition in the
             Proof Table."
2291.4   S   :CLICK-OK
2292.3   T   "It's your turn to do this step."
2294.6   S   (:MOUSE-CLICK (:PARALLEL (:S E H) (:S F G)))
2294.9   T   "Good.  Go on to the next step."
2302.2   S   :CLICK-OK
2303.6   T   "Justify a given proposition"
2305.2   S   :CLICK-OK
2305.9   T   "Now, it's your turn.  Please enter a justification of the
             postulate FG // EH into the highlighted cell."
2325.3   S   (:STRING-S TRANS)
2325.6   T   "It's not what I expect to hear from you.  The justification of
             the proposition FG // EH is GIVEN.  Enter it into the Proof
             Table."
2355.9   S   :CLICK-OK
2356.5   T   "Now, it's your turn.  Please enter a justification of the
             postulate FG // EH into the highlighted cell."
2365.9   S   (:STRING-S GIVEN)
2366.3   T   "Good.  Go on to the next step."
2367.6   S   :CLICK-OK
2368.9   T   "Let's make one step toward a complete proof. Namely, justify an
             unjustified proposition in the Proof Table."
2369.6   S   :CLICK-OK
2370.2   T   "This is done by 2 steps, which are [Select proposition to
             justify] and [Justify given proposition]."
2370.6   S   :CLICK-OK
2371.2   T   "The first step is to specify an unjustified proposition in the
             Proof Table."
2371.9   S   :CLICK-OK
2372.8   T   "It's your turn to do this step."
2374.1   S   (:MOUSE-CLICK (:EQ (:S B F) (:S F C)))
2374.4   T   "Good.  Go on to the next step."
2376.9   S   :CLICK-OK
```

```
2378.3   T   "Justify a given proposition"
2378.6   S   :CLICK-OK
2379.2   T   "Now, it's your turn.  Please enter a justification of the
             postulate BF = CF into the highlighted cell."
2387.4   S   (:STRING-S GIVEN)
2387.7   T   "Good.  Go on to the next step."
2390.0   S   :CLICK-OK
2391.3   T   "Let's make one step toward a complete proof. Namely, justify an
             unjustified proposition in the Proof Table."
2392.0   S   :CLICK-OK
2392.7   T   "This is done by 2 steps, which are [Select proposition to
             justify] and [Justify given proposition]."
2393.3   S   :CLICK-OK
2393.9   T   "The first step is to specify an unjustified proposition in the
             Proof Table."
2394.4   S   :CLICK-OK
2395.3   T   "It's your turn to do this step."
2396.9   S   (:MOUSE-CLICK (:EQ (:S D G) (:S G C)))
2397.2   T   "Good.  Go on to the next step."
2400.6   S   :CLICK-OK
2402.0   T   "Justify a given proposition"
2402.6   S   :CLICK-OK
2403.2   T   "Now, it's your turn.  Please enter a justification of the
             postulate CG = DG into the highlighted cell."
2413.3   S   (:STRING-S GIVEN)
2413.7   T   "Good.  Go on to the next step."
2416.3   S   :CLICK-OK
2417.0   T   "Good job!!!"
2430.6   S   :CLICK-OK
2431.2   T   "We've complete the proof.  Now, go ahead to the next problem.
```

APPENDIX C: The Geometry Booklet (Backward Tutor)

The booklet used for the backward chaining tutor condition.

Things to Learn with the Geometry Tutor



The Advanced Geometry Tutor Project

Learning Research and Development Center

University of Pittsburgh

May 17, 2004 / BC

1. What is a proof and how it is written?

A proof is a logical explanation of why some *proposition* is true. In geometry, a proposition can be true when it is a *fact* or it is justified by a *postulate*. Hence, a proof should read either like "A proposition X is true because it's a fact" or "A proposition X is true because a postulate Y holds."

Proposition: A statement showing a relationship of two geometric objects. For example, AB = AC.

A postulate has one or more *premises* and a *consequence*. A postulate is *held* when all the premises are true. If a postulate is held, then its consequence is said to be *justified*. The postulate is also called a *justification* of the consequence.

Postulate: A statement that is believed to be true. For example, "two base angles of an isosceles triangle are congruent."

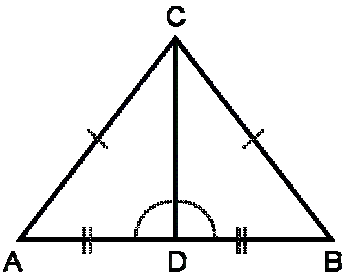
A fact is a proposition that is either *given* or very obvious. In this course, the only obvious fact is an *identical* proposition such like AB = AB; namely, every geometric object is congruent (or equivalent) to itself.

Section 2 (p.4) shows all postulates that you will learn with the Geometry Tutor

In sum, to write a proof of a proposition is to justify that the proposition is true. The justification is done either by stating that the proposition is a fact or showing that a postulate that concludes with the target proposition is held. To show a postulate is held, it is also needed to show that all its premises are justified. Hence, a proof might be long.

In actual problems you solve, a proposition for which you need to write a proof is specified as a *goal*. True propositions are called *givens*. Here is an example of a problem and its proof:

Given: AD = BD
∠CDA = ∠CDB
Goal: AC = BC



This is the Proof Table you need to complete.

The name of a postulate. See "Things to Learn" booklet for their details.

The line numbers where the premise of this postulate is shown.

	Proposition	Justification
1.	AC = BC	CPCTC (2)
2.	△CAD≅△CBD	SAS (3, 4, 5)
3.	CD = CD	Identical
4.	AD = BD	Given
5.	∠CDA = ∠CDB	Given

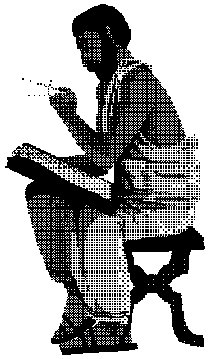
The goal to prove appears in the 1st row.

This whole line means that △CAD≅△CBD is justified with the postulate SAS given that the postulates 3, 4, and 5 can be also justified.

For an identical proposition, state "Identical" as its justification.

The line numbers

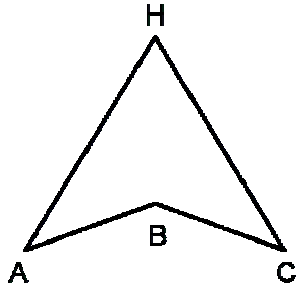
Given propositions are justified as "Given."



For some problems, you may need to draw additional segments on a given problem figure. This is called *construction*.

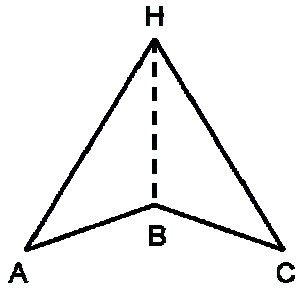
A problem shown below is one that requires construction.

Given: AH = CH
AB = CB
Goal: ∠HAB = ∠HCB



To apply CPCTC (corresponding parts of congruent triangles are congruent) to justify ∠HAB = ∠HCB, you need to draw segment HB so that you have two triangles ΔHAB and ΔHCB in the resulting figure.

Given: AH = CH
AB = CB
Goal: ∠HAB = ∠HCB



	Proposition	Justification
1.	∠HAB ≅ ∠HCB	CPCTC (2)
2.	ΔAHB ≅ ΔCHB	SSS (3, 4, 5)
3.	HB = HB	Identical
4.	AB = CB	Given
5.	AH = HC	Given

2. The Postulates You Need to Know to Study with the Geometry Tutor

We will learn 10 postulates that you can use in your proof. This section shows all of the 10 postulates. You can use this booklet as a reference when you write a proof. Each postulate first shows a brief description. It then shows a configuration that best represents the postulate. Given a configuration, the premises and the consequences of the postulate are then shown so that you can use the postulate in your proof.

2.1. Corresponding Parts Postulate (CPCTC)

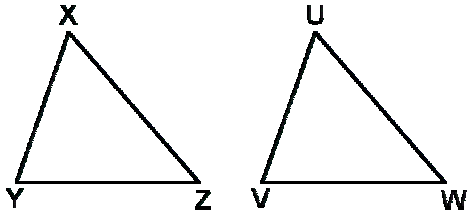
One of the most common ways to justify a proposition about equality is to claim that they are a part of two congruent triangles. CPCTC is the postulate that supports this claim.

Description:

Corresponding parts of two *congruent triangles* are *congruent*

Configuration:

ΔXYZ and ΔUVW



Premises:

$\Delta XYZ \equiv \Delta UVW$

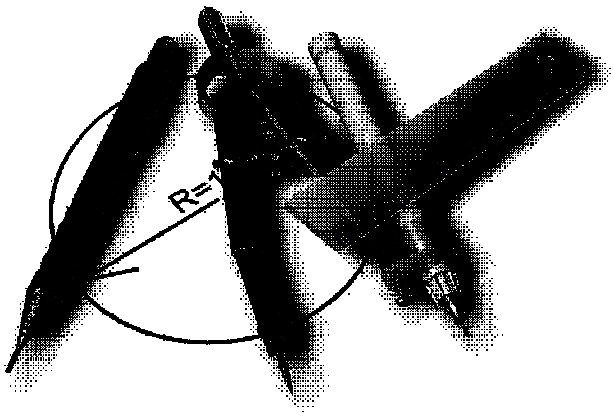
Consequence:

$XY = UV$, $YZ = VW$, $XZ = UW$,
$\angle YXZ = \angle VUW$, $\angle XYZ = \angle UVW$, and $\angle XZY = \angle UWV$

Corresponding parts: If two triangles can be perfectly overlapped, then each overlapping apex, segment, and angle are called corresponding parts

Congruent triangles: Two triangles are congruent if they can be overlapped completely.

Congruent: In general two geometric objects are congruent if they can be overlapped completely.



2.2. Side-Angle-Side Postulate (SAS)

There are three postulates for triangle congruence. That is there are three ways to justify that two triangles are congruent. Here is the first postulate.

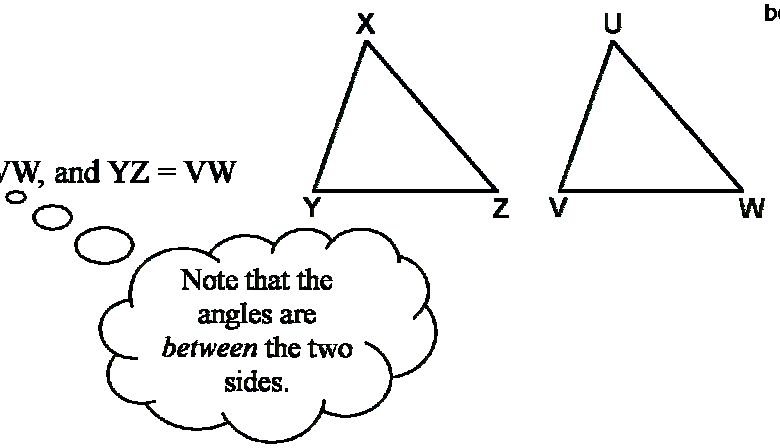
Description:

> If two sides and the *angle between them* in one triangle are congruent to the corresponding parts in the other triangle, then the triangles are congruent.

An angle between two sides: Every angle has two sides. The angle is called to be an angle between those two sides. For example, in the left figure, ∠XYZ is an angle between sides XY and YZ

Configuration:

> ΔXYZ and ΔUVW



Premises:

> XY = UV, ∠XYZ = ∠UVW, and YZ = VW

Consequence:

> ΔXYZ ≡ ΔUVW

2.3. Side-Side-Side Postulate (SSS)

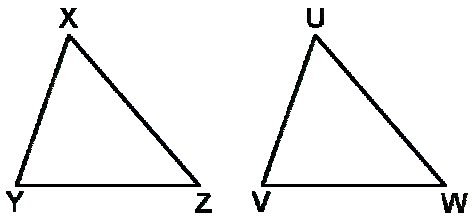
This is the second postulate for triangle congruence.

Description:

> If all three sides in one triangle are congruent to the corresponding sides in the other triangle, then the two triangles are congruent.

Configuration:

> ΔXYZ and ΔUVW



Premises:

> XY = UV, YZ = VW, and ZX = WU

Consequence:

> ΔXYZ ≡ ΔUVW

2.4. Angle-Side-Angle Postulate (ASA)

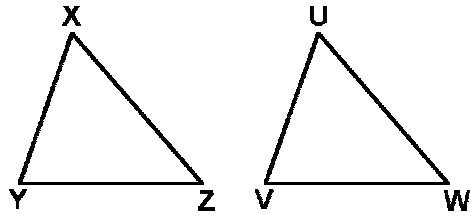
Here is the third and the last postulate for triangle congruence.

Description:

If two angles and the side between them in one triangle are congruent to the corresponding parts in another triangle, then the triangles are congruent.

Configuration:

ΔXYZ and ΔUVW



Premises:

$\angle XYZ = \angle UVW$, $YZ = VW$, and $\angle XZY = \angle UWV$

Consequence:

$\Delta XYZ \equiv \Delta UVW$

Note that the angles are on the both sides of YZ and VW.

2.5. Alternate Interior Angle Congruence (Z)

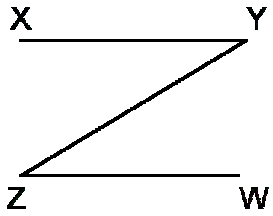
When parallel lines meet with another line, there is a pair of angles that are congruent.

Description:

Given two lines cut by a *transversal*. If the lines are *parallel*, then a pair of *alternative interior angles* is congruent.

Configuration:

A line XY, YZ, and ZW



Premises:

$XY \parallel ZW$

Consequence:

$\angle XYZ = \angle YZW$

Transversal: A transversal of two lines is a line that intersects them at different points.

Parallel: Two lines that never intersect are called parallel lines.

Alternative Interior Angles: Given two lines XY and ZW. The segment YZ is called *transversal*. Now, $\angle XYZ$ and $\angle YZW$ are called alternative interior angles.

So, notice that the Z postulate says that "the alternative interior angles made by parallel lines are congruent."

2.6. Triangle Midpoint Theorem (TriM)

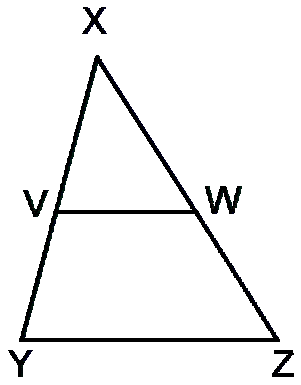
Description:

Given a triangle, a segment connecting *midpoints* of two sides is parallel to the other side.

Midpoint: A point that divides a segment into two equal segments is called a midpoint.

Configuration:

ΔXYZ with a point V on XY and W on XZ



Premises:

XV = VY and XW = WZ

Consequence:

VW // YZ.

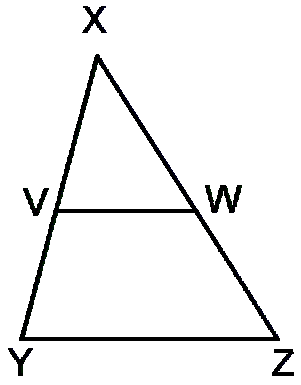
2.7. Inverse of Triangle Midpoint Theorem (MTri)

Description:

If a segment that intersects with a side of a triangle at its midpoint is parallel to another side, then the segment bisect the third segment.

Configuration:

ΔXYZ with a point V on XY and W on XZ

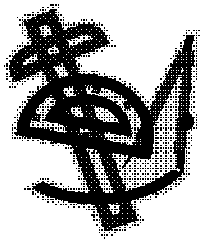


Premises:

XV = VY and VW // YZ

Consequence:

XW = WZ



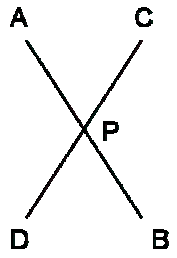
2.8. Vertical Angle Postulate (VerAng)

When two segments meet, there are always two pairs of angles facing each other, which are called *vertical angles*.

Vertical angles: Two angles are vertical angles if their sides form two pairs of opposite rays.

Description:

Vertical angles are congruent.



Configuration:

Two segments AB and CD intersecting at P

Premises:

There are no premises needed to justify this postulate. When you see two segments intersecting like the one in the above figure, then you can immediately conclude the following consequences.

Consequence:

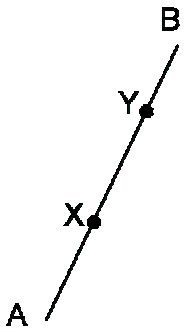
$\angle APC = \angle BPD$ and $\angle APD = \angle BPC$

2.9. Collinear Parallel (COLPARA)

This postulate may sound silly, but to make a valid proof, you may need this.

Description:

Two overlapping segments are parallel.



Configuration:

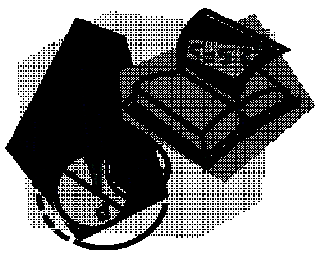
Segments AB and XY, overlapping

Premises:

There is no premises need to justify for this postulate.

Consequence

AB // XY



2.10. Transitive Substitution (Trans)

This postulate can be applied to congruent segments, angles, and triangles, as well as parallel lines. The premise and the consequence sections below show both.

Description:

If two propositions about segment congruence share the same segment, then the other two segments involved in the propositions are also congruent.

You can substitute "segment" with "angle" or "triangle."

If two propositions about parallel segments share the same segment, then other two segments involved the propositions are also parallel.

Configuration:

No configuration is associated with this postulate.

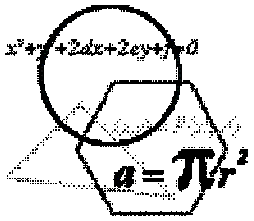
Premises:

AB = XY and XY = CD AB // XY and XY // CD

XY is the shared segment

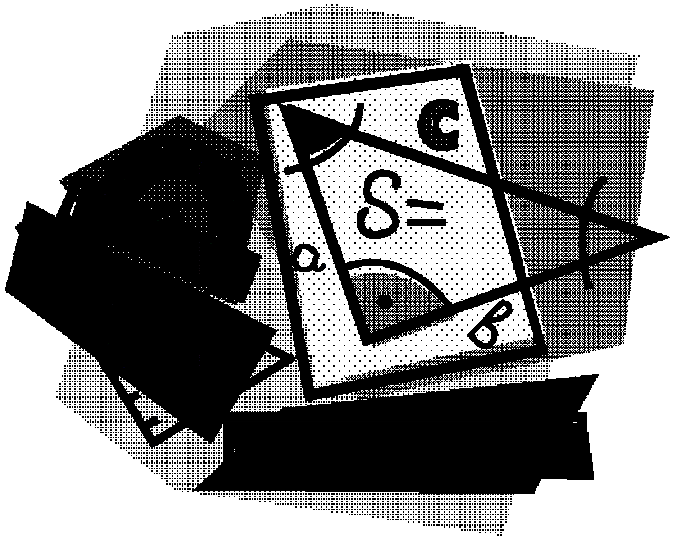
Consequence

AB = CD AB // CD



APPENDIX D: The Geometry Booklet (Forward Tutor)

Things to Learn with the Geometry Tutor



The Advanced Geometry Tutor Project

Learning Research and Development Center

University of Pittsburgh

May 17, 2004 / FC

1. What is a proof and how it is written?

A proof is a logical explanation of why some *proposition* is true. In geometry, a proposition can be true when it is a *fact* or it is justified by a *postulate*. Hence, a proof should read either like "A proposition X is true because it's a fact" or "A proposition X is true because a postulate Y holds."

Proposition: A statement showing a relationship of two geometric objects. For example, AB = AC.

A postulate has one or more *premises* and a *consequence*. A postulate is *held* when all the premises are true. If a postulate is held, then its consequence is said to be *justified*. The postulate is also called a *justification* of the consequence.

Postulate: A statement that is believed to be true. For example, "two base angles of an isosceles triangle are congruent."

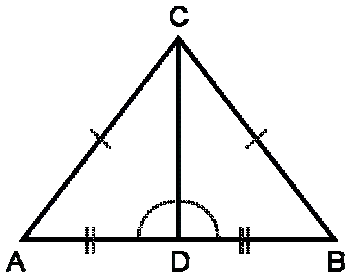
A fact is a proposition that is either *given* or very obvious. In this course, the only obvious fact is an *identical* proposition such like AB = AB; namely, every geometric object is congruent (or equivalent) to itself.

Section 2 (p.4) shows all postulates that you will learn with the Geometry Tutor

In sum, to write a proof of a proposition is to justify that the proposition is true. The justification is done either by stating that the proposition is a fact or showing that a postulate that concludes with the target proposition is held. To show a postulate is held, it is also needed to show that all its premises are justified. Hence, a proof might be long.

In actual problems you solve, a proposition for which you need to write a proof is specified as a *goal*. True propositions are called *givens*. Here is an example of a problem and its proof:

Given: AD = BD
∠CDA = ∠CDB
Goal: AC = BC



This is the Proof Table you need to complete.

A given proposition must be justified as "Given."

	Proposition	Justification
1.	AD = BD	Given
2.	∠CDA = ∠CDB	Given
3.	CD = CD	Identical
4.	△CAD≡△CBD	SAS (1,2,3)
5.	AC = BC	CPCTC (4)

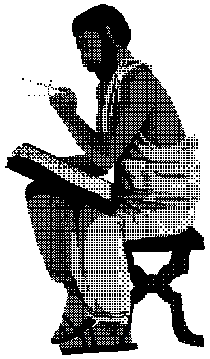
For an identical proposition, state "Identical" as its justification.

This whole line means that △CAD≡△CBD is justified with the postulate SAS given that the postulates 1, 2, and 3 are also justified.

The line numbers

The name of a postulate. See "Things to Learn" booklet for their details.

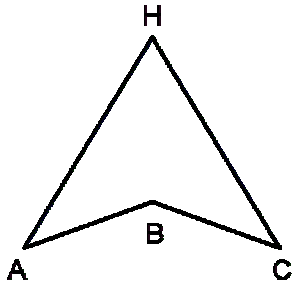
The line numbers where each premise is shown.



For some problems, you may need to draw additional segments on a given problem figure. This is called *construction*.

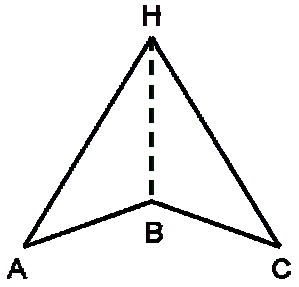
A problem shown below is one that requires construction.

Given: AH = CH
AB = CB
Goal: ∠HAB = ∠HCB



To apply CPCTC (corresponding parts of congruent triangles are congruent) to justify ∠HAB = ∠HCB, you need to draw segment HB so that you have two triangles ΔHAB and ΔHCB in the resulting figure.

Given: AH = CH
AB = CB
Goal: ∠HAB = ∠HCB



	Proposition	Justification
1.	AH = HC	Given
2.	AB = CB	Given
3.	HB = HB	Identical
4.	ΔAHB ≅ ΔCHB	SSS (1, 2, 3)
5.	∠HAB = ∠HCB	CPCTC (4)

2. The Postulates You Need to Know to Study with the Geometry Tutor

We will learn 10 postulates that you can use in your proof. This section shows all of the 10 postulates. You can use this booklet as a reference when you write a proof. Each postulate first shows a brief description. It then shows a configuration that best represents the postulate. Given a configuration, the premises and the consequences of the postulate are then shown so that you can use the postulate in your proof.

2.1. Corresponding Parts Postulate (CPCTC)

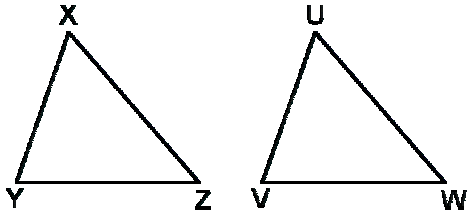
One of the most common ways to justify a proposition about equality is to claim that they are a part of two congruent triangles. CPCTC is the postulate that supports this claim.

Description:

Corresponding parts of two *congruent triangles* are *congruent*

Configuration:

ΔXYZ and ΔUVW



Premises:

$\Delta XYZ \equiv \Delta UVW$

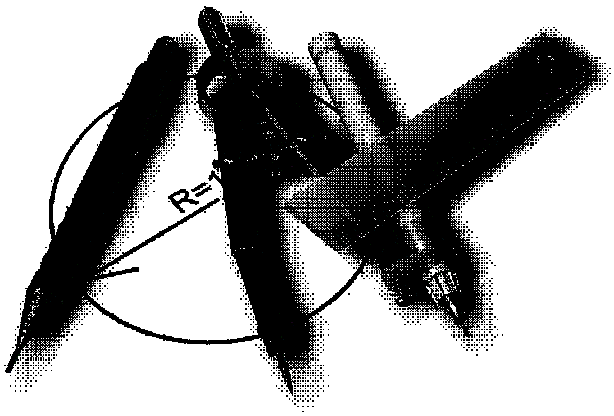
Consequence:

$XY = UV$, $YZ = VW$, $XZ = UW$,
$\angle YXZ = \angle VUW$, $\angle XYZ = \angle UVW$, and $\angle XZY = \angle UWV$

Corresponding parts: If two triangles can be perfectly overlapped, then each overlapping apex, segment, and angle are called corresponding parts

Congruent triangles: Two triangles are congruent if they can be overlapped completely.

Congruent: In general two geometric objects are congruent if they can be overlapped completely.



2.2. Side-Angle-Side Postulate (SAS)

There are three postulates for triangle congruence. That is there are three ways to justify that two triangles are congruent. Here is the first postulate.

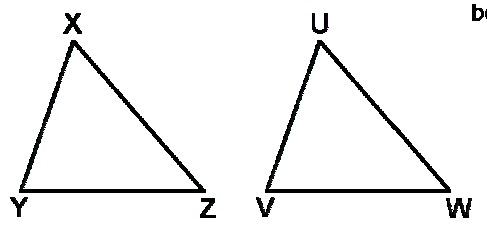
Description:

> If two sides and the *angle between them* in one triangle are congruent to the corresponding parts in the other triangle, then the triangles are congruent.

An angle between two sides: Every angle has two sides. The angle is called to be an angle between those two sides. For example, in the left figure, ∠XYZ is an angle between sides XY and YZ

Configuration:

> ΔXYZ and ΔUVW



Premises:

> XY = UV, ∠XYZ = ∠UVW, and YZ = VW

Consequence:

> ΔXYZ ≡ ΔUVW

Note that the angles are *between* the two sides.

2.3. Side-Side-Side Postulate (SSS)

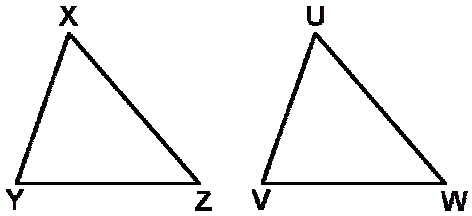
This is the second postulate for triangle congruence.

Description:

> If all three sides in one triangle are congruent to the corresponding sides in the other triangle, then the two triangles are congruent.

Configuration:

> ΔXYZ and ΔUVW



Premises:

> XY = UV, YZ = VW, and ZX = WU

Consequence:

> ΔXYZ ≡ ΔUVW

2.4. Angle-Side-Angle Postulate (ASA)

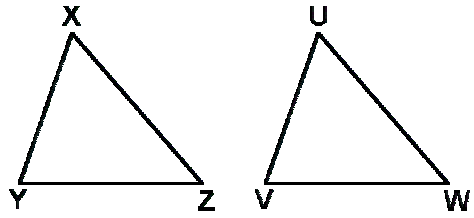
Here is the third and the last postulate for triangle congruence.

Description:

If two angles and the side between them in one triangle are congruent to the corresponding parts in another triangle, then the triangles are congruent.

Configuration:

ΔXYZ and ΔUVW



Premises:

$\angle XYZ = \angle UVW$, $YZ = VW$, and $\angle XZY = \angle UWV$

Consequence:

$\Delta XYZ \equiv \Delta UVW$

Note that the angles are on the both sides of YZ and VW.

2.5. Alternate Interior Angle Congruence (Z)

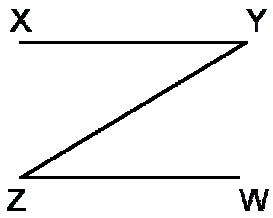
When parallel lines meet with another line, there is a pair of angles that are congruent.

Description:

Given two lines cut by a *transversal*. If the lines are *parallel*, then a pair of *alternative interior angles* is congruent.

Configuration:

A line XY, YZ, and ZW



Premises:

$XY \parallel ZW$

Consequence:

$\angle XYZ = \angle YZW$

Transversal: A transversal of two lines is a line that intersects them at different points.

Parallel: Two lines that never intersect are called parallel lines.

Alternative Interior Angles: Given two lines XY and ZW. The segment YZ is called *transversal*. Now, $\angle XYZ$ and $\angle YZW$ are called alternative interior angles.

So, notice that the Z postulate says that "the alternative interior angles made by parallel lines are congruent."

2.6. Triangle Midpoint Theorem (TriM)

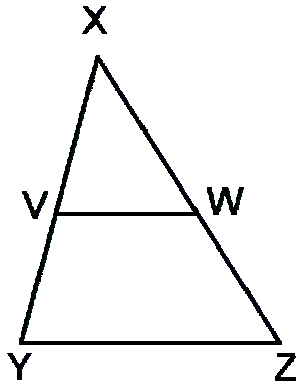
Description:

Given a triangle, a segment connecting *midpoints* of two sides is parallel to the other side.

Midpoint: A point that divides a segment into two equal segments is called a midpoint.

Configuration:

ΔXYZ with a point V on XY and W on XZ



Premises:

XV = VY and XW = WZ

Consequence:

VW // YZ.

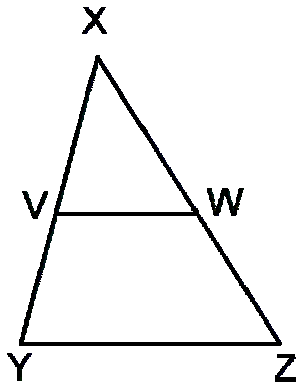
2.7. Inverse of Triangle Midpoint Theorem (MTri)

Description:

If a segment that intersects with a side of a triangle at its midpoint is parallel to another side, then the segment bisect the third segment.

Configuration:

ΔXYZ with a point V on XY and W on XZ

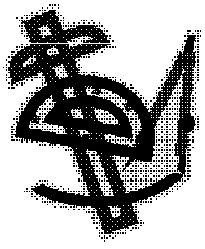


Premises:

XV = VY and VW // YZ

Consequence:

XW = WZ



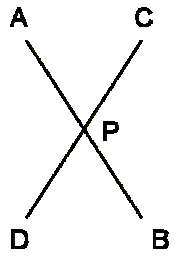
2.8. Vertical Angle Postulate (VerAng)

When two segments meet, there are always two pairs of angles facing each other, which are called *vertical angles*.

Vertical angles: Two angles are vertical angles if their sides form two pairs of opposite rays.

Description:

Vertical angles are congruent.



Configuration:

Two segments AB and CD intersecting at P

Premises:

There are no premises needed to justify this postulate. When you see two segments intersecting like the one in the above figure, then you can immediately conclude the following consequences.

Consequence:

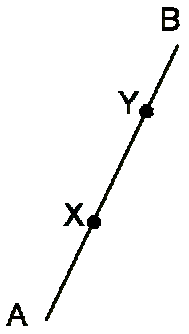
$\angle APC = \angle BPD$ and $\angle APD = \angle BPC$

2.9. Collinear Parallel (COLPARA)

This postulate may sound silly, but to make a valid proof, you may need this.

Description:

Two overlapping segments are parallel.



Configuration:

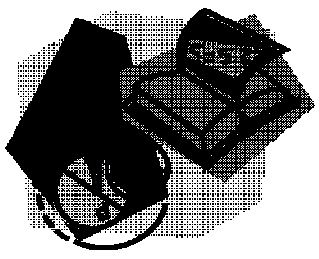
Segments AB and XY, overlapping

Premises:

There is no premises need to justify for this postulate.

Consequence

AB // XY



2.10. Transitive Substitution (Trans)

This postulate can be applied to congruent segments, angles, and triangles, as well as parallel lines. The premise and the consequence sections below show both.

Description:

If two propositions about segment congruence share the same segment, then the other two segments involved in the propositions are also congruent.

You can substitute "segment" with "angle" or "triangle."

If two propositions about parallel segments share the same segment, then other two segments involved the propositions are also parallel.

Configuration:

No configuration is associated with this postulate.

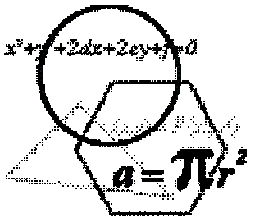
Premises:

AB = XY and XY = CD AB // XY and XY // CD

XY is the shared segment

Consequence

AB = CD AB // CD



APPENDIX E: Test-A (Backward Tutor)

Geometry Theorem Proving Test (A)

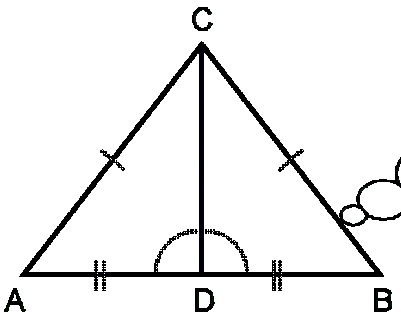
ID: BC-_____ Pre / Post Date: ____/_____/ 2004

For each problem write a proof of the goal using the postulates shown in the *Things to Learn* booklet.

Each row of the proof must have a proposition in the left column and a justification in the right column. Specify the premises of a justification by a list of line numbers where the premises are located. Use the givens as true propositions.

Here is an example problem and its proof:

Given: AD = BD
∠CDA = ∠CDB
Goal: AC = BC



Feel free to put marks on the figure. But, please DO NOT erase them.

Proof:

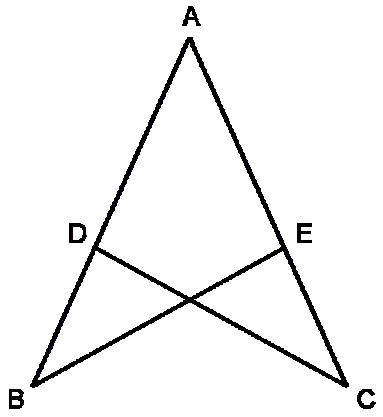
	Proposition	Justification
1	AC = BC	CPCTC (2)
2	ΔCAD ≡ ΔCBD	SAS (3, 4, 5)
3	CD = CD	Identity
4	AD = BD	Given
5	∠CDA = ∠CDB	Given

Some tips:

- For propositions that are given, write "Given" as its justification.
- For propositions for equality of two identical things (e.g., CD = CD), write "Identical" as its justification.
- For other justifications, write a name of the postulate followed by a list of line numbers showing premises.

1. Complete a proof for the goal by filling the blanks with an appropriate postulate name

Given: AB = AC
$\angle ABE = \angle ACD$
Goal: BE = CD

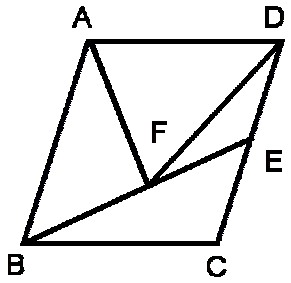


Proof:

	Proposition	Justification
1	BE = CD	_____ (2)
2	$\Delta ABE \equiv \Delta ACD$	ASA (3, 4, 5)
3	$\angle ABE = \angle ACD$	Given
4	AB = AC	Given
5	$\angle BAE = \angle CAD$	_____

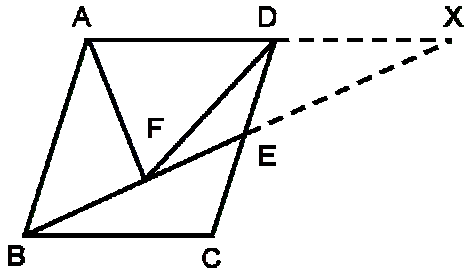
2. Complete a proof for the goal by filling the blanks with an appropriate postulate name

Given: AD // BC
AD = BC
AF ⊥ BE
CE = ED
Goal: AD = FD



Proof:

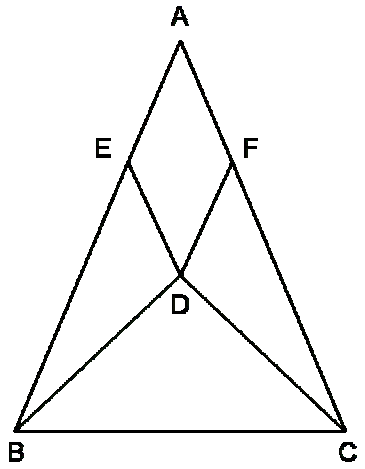
This problem needs a construction to complete the proof. I extend segment AD and segment BE and plot a point X as their intersection. Notice that AD // BC implies that AX // BC. Also notice that AF ⊥ BE implies AF ⊥ FX.



	Proposition	Justification
1	AD = FD	Right Angle Triangle Midpoint Theorem (2, 3)
2	AD = DX	Trans (4, 5)
3	AF ⊥ FX	Given
4	AD = BC	Given
5	BC = DX	CPCTC (6)
6	ΔBCE ≡ ΔXDE	_____ (7, 8, 9)
7	∠BEC = ∠XED	_____
8	CE = ED	Given
9	∠BCE = ∠XDE	_____ (10)
10	AX // BC	Given

3. Complete a proof for the goal by filling the blanks with an appropriate postulate name

Given: AB = AC
EB = FC
DB = DC
Goal: ED = FD

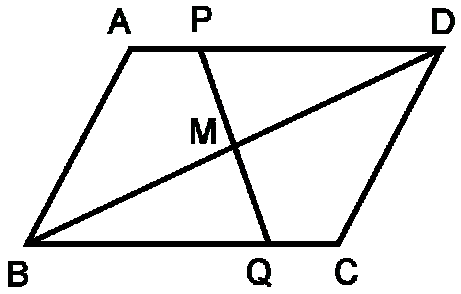


Proof:

	Proposition	Justification
1	ED = FD	CPCTC (2)
2	ΔEBD ≡ ΔFCD	_____ (3, 4, 5)
3	EB = FC	Given
4	∠EBD = ∠FCD	Differences of equal angles (6, 7)
5	DB = DC	Given
6	∠ABC = ∠ACB	Base angles of an isosceles triangle are equal (8)
7	∠DBC = ∠DCB	Base angles of an isosceles triangle are equal (5)
8	AB = AC	Given

4. Write a proof for the goal

Given: AD // BC
AB // DC
BM = MD
Goal: PM = QM

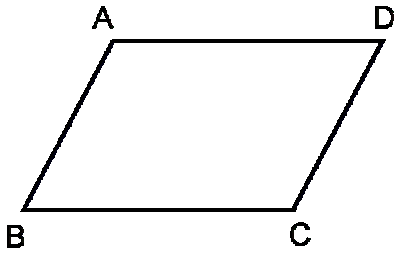


Proof:

	Proposition	Justification
1.	PM = QM	

5. Write a proof for the goal

Given: BC = AD
BC // AD
Goal: AB = CD

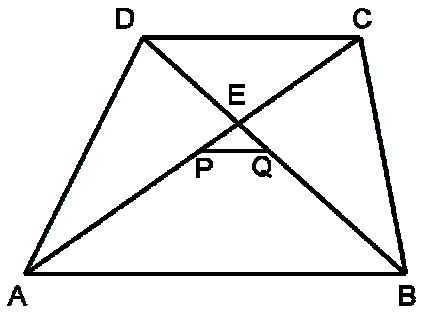


Proof:

	Proposition	Justification
1.	AB = CD	

6. Write a proof for the goal

Given: PQ // AB
CD // AB
AP = PC
Goal: DQ = QB



Proof:

	Proposition	Justification
1.	DQ = QB	

APPENDIX F: Test-B (Backward Tutor)

Geometry Theorem Proving Test (B)

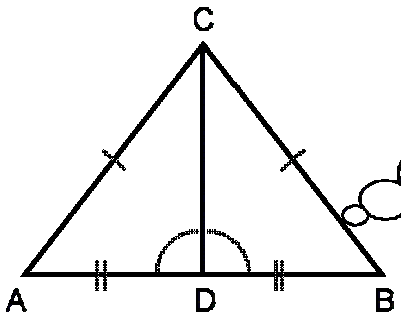
ID: BC-_____ Pre / Post Date: ____/_____/ 2004

For each problem write a proof of the goal using the postulates shown in the *Things to Learn* booklet.

Each row of the proof must have a proposition in the left column and a justification in the right column. Specify the premises of a justification by a list of line numbers where the premises are located. Use the givens as true propositions.

Here is an example problem and its proof:

Given: AD = BD
∠CDA = ∠CDB
Goal: AC = BC



Feel free to put marks on the figure. But, please DO NOT erase them.

Proof:

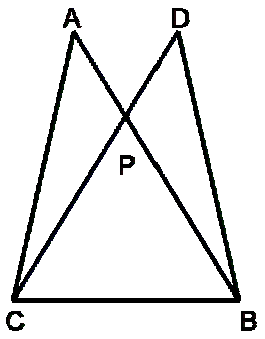
	Proposition	Justification
1	AC = BC	CPCTC (2)
2	ΔCAD ≡ ΔCBD	SAS (3, 4, 5)
3	CD = CD	Identical
4	AD = BD	Given
5	∠CDA = ∠CDB	Given

Some tips:

- For propositions that are given, write "Given" as its justification.
- For propositions for equality of two identical things (e.g., CD = CD), write "Identical" as its justification
- For other justifications, write a name of the postulate followed by a list of line numbers showing premises

1. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AB = CD
$\angle DCB = \angle ABC$
Goal: AC = DB

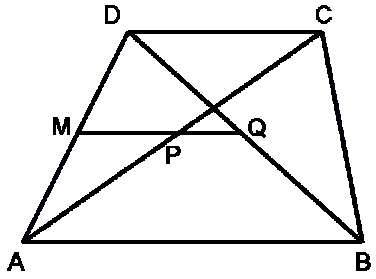


Proof:

	Proposition	Justification
1	AC = DB	_____ (2)
2	$\Delta ACB \equiv \Delta DBC$	SAS (3, 4, 5)
3	AB = AC	Given
4	$\angle ABC = \angle DCB$	Given
5	CB = BC	_____

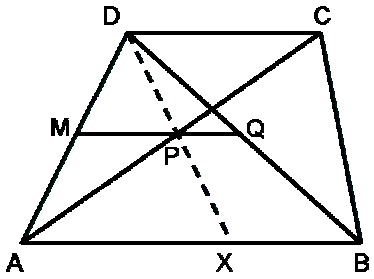
2. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: CD // AB
AP = PC
MQ // AB
Goal: AM = MD



Proof:

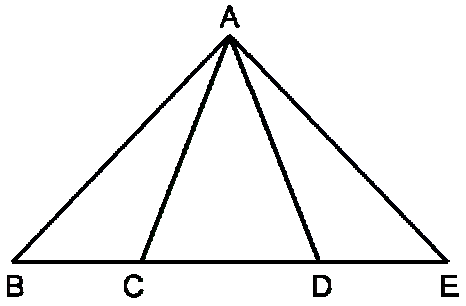
This problem needs a construction to complete the proof. I connect points D and P, extend it to the segment AB, and plot X as their intersection.



	Proposition	Justification
1	AM = MD	MTri (2, 3)
2	MQ // AB	Given
3	DP = PX	CPCTC (4)
4	ΔDPC ≡ ΔXPA	_____ (5, 6, 7)
5	AP = PC	Given
6	∠DCP = ∠XAP	_____ (8)
7	∠DPC = ∠XPA	_____
8	CD // AB	Given

3. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AC = AD
BC = ED
Prove: AB = AE

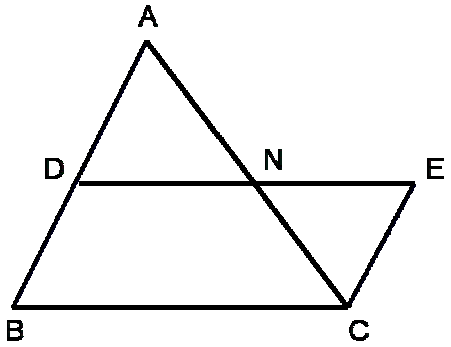


Proof:

	Proposition	Justification
1	AB = AE	CPCTC (2)
2	ΔABC ≡ ΔAED	_____ (3, 4, 5)
3	AC = AD	Given
4	∠ACB = ∠ADE	Differences of equal angles (6, 7)
5	BC = ED	Given
6	∠BCD = ∠EDC	Both angles are 180° hence equal
7	∠ACD = ∠ADC	Base angles of an isosceles triangle are equal (3)

4. Write a proof for the goal

Given: AB // EC
AN = NC
Goal: AD = EC

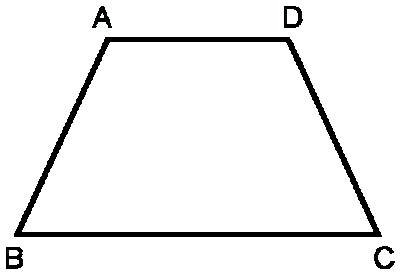


Proof:

	Proposition	Justification
1.	AD = EC	

5. Write a proof for the goal

Given: AB = DC
∠ABC = ∠DCB
Goal: ∠BAD = ∠CDA

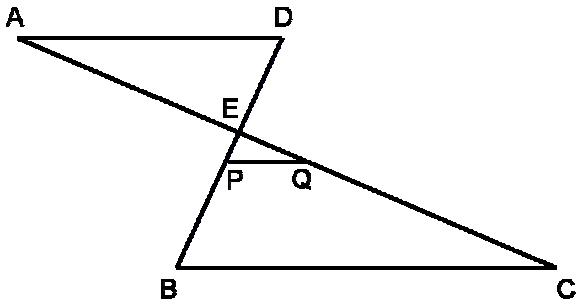


Proof:

	Proposition	Justification
1.	∠BAD = ∠CDA	

6. Write a proof for the goal

Given: DP = PB
AD // BC
PQ // BC
Goal: AQ = QC



Proof:

	Proposition	Justification
1.	AQ = QC	

APPENDIX G: Test-A (Forward Tutor)

Form: 2004-05-12

Geometry Theorem Proving Test (A)

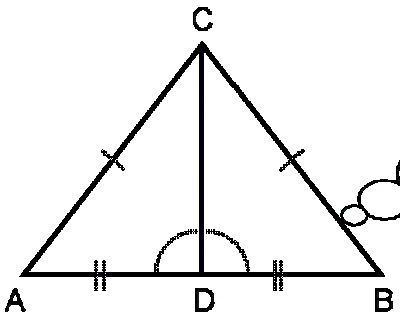
ID: FC-_____ Pre / Post Date: ____/_____/ 2004

For each problem write a proof of the goal using the postulates shown in the *Things to Learn* booklet.

Each row of the proof must have a proposition in the left column and a justification in the right column. Specify the premises of a justification by a list of line numbers where the premises are located. Use the givens as true propositions.

Here is an example problem and its proof:

Given: AD = BD
∠CDA = ∠CDB
Goal: AC = BC



Feel free to put marks on the figure. But, please DO NOT erase them.

Proof:

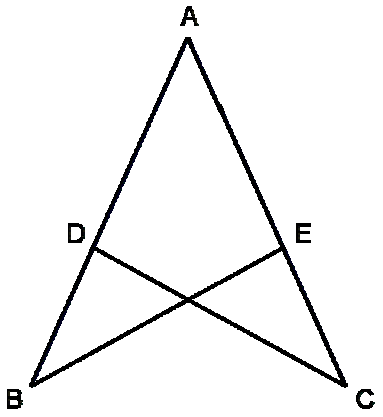
	Proposition	Justification
1	AD = BD	Given
2	∠CDA = ∠CDB	Given
3	CD = CD	Identical
4	ΔCAD ≡ ΔCBD	SAS (1, 2, 3)
5	AC = BC	CPCTC (4)

Some tips:

- For propositions that are given, write "Given" as its justification.
- For propositions for equality of two identical things (e.g., CD = CD), write "Identical" as its justification.
- For other justifications, write a name of the postulate followed by a list of line numbers showing premises.

1. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AB = AC
∠ABE = ∠ACD
Goal: BE = CD

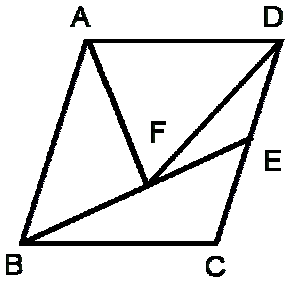


Proof:

	Proposition	Justification
1	AB = AC	Given
2	∠ABE = ∠ACD	Given
3	∠BAE = ∠CAD	_____
4	ΔABE ≡ ΔACD	ASA (1, 2, 3)
5	BE = CD	_____ (4)

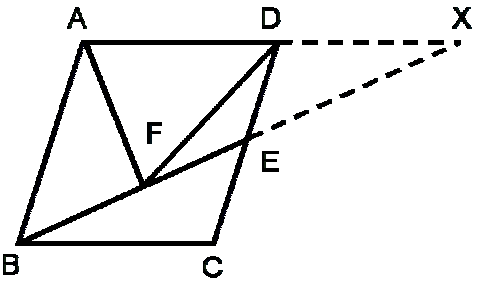
2. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AD // BC
AD = BC
AF ⊥ BE
CE = ED
Goal: AD = FD



Proof:

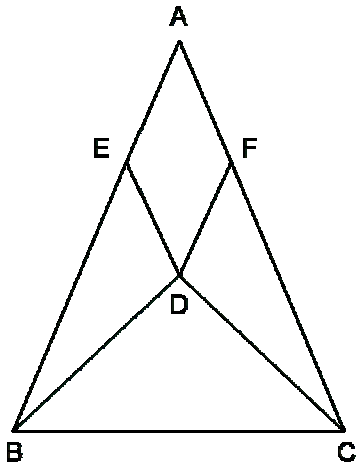
This problem needs a construction to complete the proof. I extend segment AD and segment BE and plot a point X as their intersection. Notice that AD // BC implies that AX // BC. Also notice that AF ⊥ BE implies AF ⊥ FX.



	Proposition	Justification
1	AX // BC	Given
2	CE = ED	Given
3	AD = BC	Given
4	AF ⊥ FX	Given
5	∠BCE = ∠XDE	_____ (1)
6	∠BEC = ∠XED	_____
7	ΔBCE ≡ ΔXDE	_____ (2, 5, 6)
8	BC = DX	CPCTC (7)
9	AD = DX	Trans (3, 8)
10	AD = FD	Right Angle Triangle Midpoint Theorem (4, 9)

3. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AB = AC
EB = FC
DB = DC
Goal: ED = FD

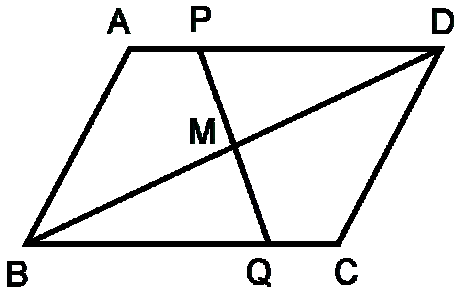


Proof:

	Proposition	Justification
1	AB = AC	Given
2	DB = DC	Given
3	EB = FC	Given
4	∠ABC = ∠ACB	Base angles of an isosceles triangle are equal (1)
5	∠DBC = ∠DCB	Base angles of an isosceles triangle are equal (2)
6	∠EBD = ∠FCD	Differences of equal angles (4, 5)
7	ΔEBD ≡ ΔFCD	_____ (2, 3, 6)
8	ED = FD	CPCTC (7)

4. Write a proof for the goal

Goal: AD // BC
AB // DC
BM = MD
Goal: PM = QM

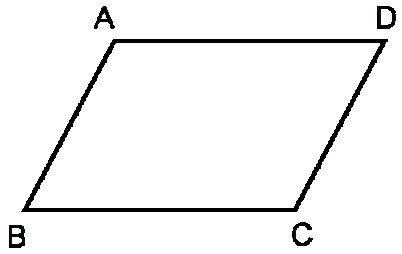


Proof:

	Proposition	Justification
1.	AD // BC	Given
2.	AB // DC	Given
3.	BM = MD	Given

5. Write a proof for the goal

Given: BC = AD
BC // AD
Goal: AB = CD

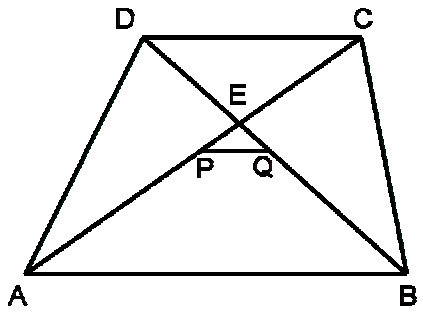


Proof:

	Proposition	Justification
1.	BC = AD	Given
2.	BC // AD	Given

6. Write a proof for the goal

Given: PQ // AB
CD // AB
AP = PC
Goal: DQ = QB



Proof:

	Proposition	Justification
1.	PQ // AB	Given
2.	CD // AB	Given
3.	AP = PC	Given

APPENDIX H: Test-B (Forward Tutor)

Form: 2004-05-12

Geometry Theorem Proving Test (B)

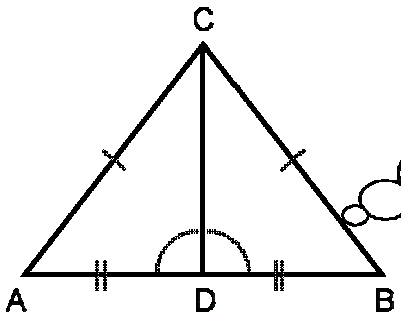
ID: FC-_____ Pre / Post Date: ____/_____/ 2004

For each problem write a proof of the goal using the postulates shown in the *Things to Learn* booklet.

Each row of the proof must have a proposition in the left column and a justification in the right column. Specify the premises of a justification by a list of line numbers where the premises are located. Use the givens as true propositions.

Here is an example problem and its proof:

Given: AD = BD
∠CDA = ∠CDB
Goal: AC = BC



Feel free to put marks on the figure. But, please DO NOT erase them.

Proof:

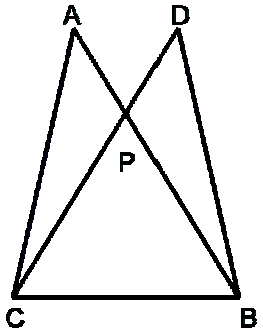
	Proposition	Justification
1	∠CDA = ∠CDB	Given
2	AD = BD	Given
3	CD = CD	Identical
4	ΔCAD ≡ ΔCBD	SAS (1, 2, 3)
5	AC = BC	CPCTC (4)

Some tips:

- For propositions that are given, write "Given" as its justification.
- For propositions for equality of two identical things (e.g., CD = CD), write "Identical" as its justification.
- For other justifications, write a name of the postulate followed by a list of line numbers showing premises.

1. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AB = CD
$\angle DCB = \angle ABC$
Goal: AC = DB

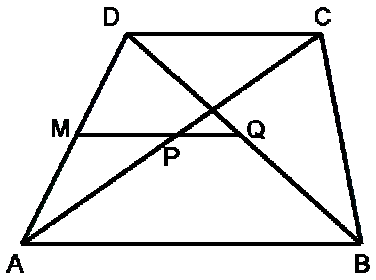


Proof:

	Proposition	Justification
1	$\angle ABC = \angle DCB$	Given
2	AB = AC	Given
3	CB = BC	_____
4	$\Delta ACB \equiv \Delta DBC$	SAS (1, 2, 3)
5	AC = DB	_____ (4)

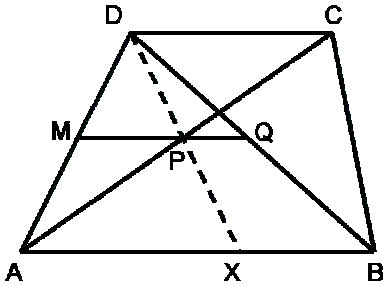
2. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: CD // AB
AP = PC
MQ // AB
Goal: AM = MD



Proof:

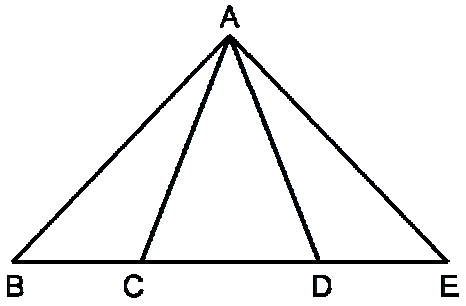
This problem needs a construction to complete the proof. I connect points D and P, extend it to segment AB, and plot X as their intersection.



	Proposition	Justification
1	CD // AB	Given
2	AP = PC	Given
3	MQ // AB	Given
4	∠DPC = ∠XPA	_____
5	∠DCP = ∠XAP	_____ (1)
6	ΔDPC ≡ ΔXPA	_____ (2, 4, 5)
7	DP = PX	CPCTC (6)
8	AM = MD	MTri (3, 7)

3. Complete a proof for the goal by filling in the blanks with an appropriate postulate name

Given: AC = AD
BC = ED
Prove: AB = AE

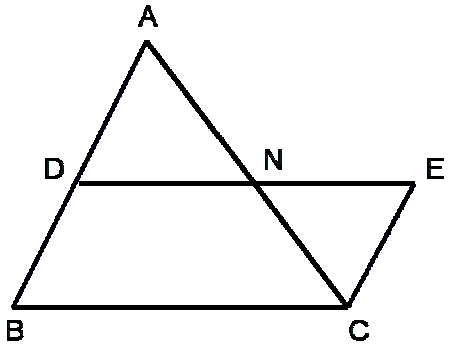


Proof:

	Proposition	Justification
1	AC = AD	Given
2	BC = ED	Given
3	∠ACD = ∠ADC	Base angles of an isosceles triangle are equal (1)
4	∠BCD = ∠EDC	Both angles are 180° hence equal
5	∠ACB = ∠ADE	Differences of the equal angles (3, 4)
6	ΔABC ≡ ΔAED	_____ (1, 2, 5)
7	AB = AE	CPCTC (6)

4. Write a proof for the goal

Given: AB // EC
AN = NC
Goal: AD = EC

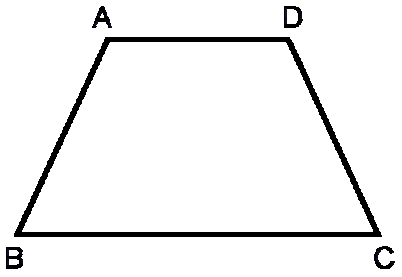


Proof:

	Proposition	Justification
1.	AB // EC	Given
2.	AN = NC	Given

5. Write a proof for the goal

Given: AB = DC
$\angle ABC = \angle DCB$
Goal: $\angle BAD = \angle CDA$

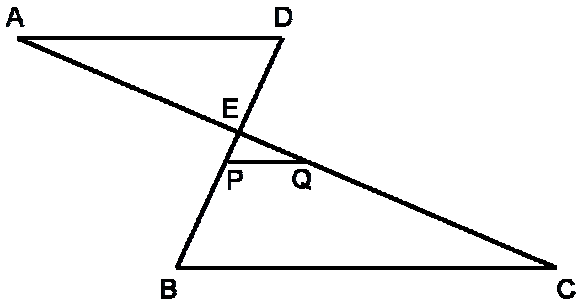


Proof:

	Proposition	Justification
1.	AB = DC	Given
2.	$\angle ABC = \angle DCB$	Given

6. Write a proof for the goal

Goal: DP = PB
AD // BC
PQ // BC
Goal: AQ = QC

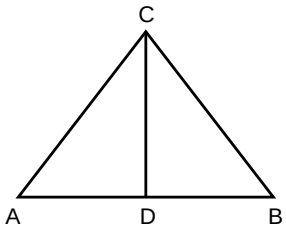


Proof:

	Proposition	Justification
1.	DP = PB	Given
2.	AD // BC	Given
3.	PQ // BC	Given

APPENDIX I: Problems used in Tutoring Sessions

#1

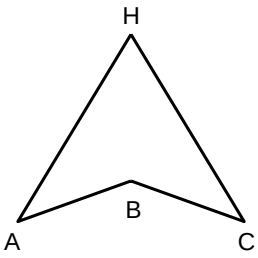


Given: AD = BD

∠CDA = ∠CDB

Goal: AC = BC

#2

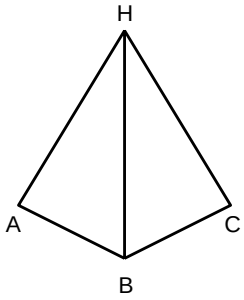


Given: AH = CH

AB = CB

Goal: ∠HAB = ∠HCB

#3

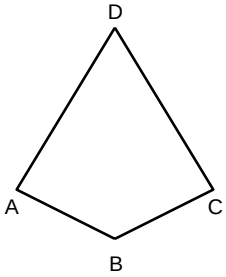


Given: AH = CH

∠AHB = ∠CHB

Goal: AB = CB

#4

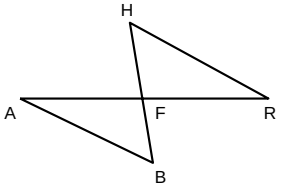


Given: AD = CD

∠ADB = ∠CDB

Goal: AB = CB

#5

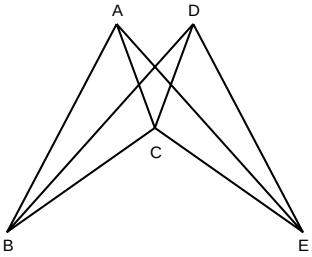


Given: AF = FR

HF = BF

Goal: AB = HR

#6



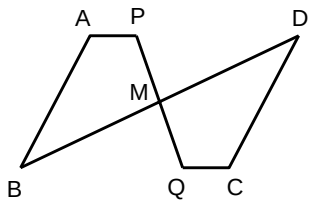
Given: AB = DE

AC = DC

BC = CE

Goal: ∠ABD = ∠DEA

#7



Goal: AP = CQ

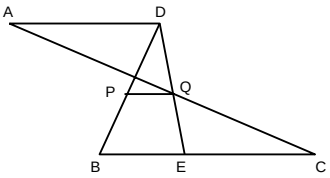
BM = MD

PM = QM

AB = DC

Goal: ∠PAB = ∠QCD

#8



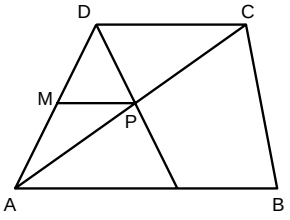
Goal: DP = PB

AD // EC

PQ // BE

Goal: AQ = QC

#9



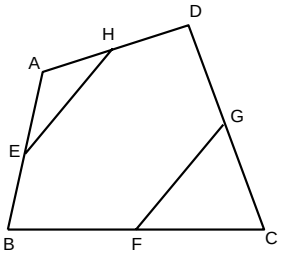
Given: AP = PC

MP // AE

CD // AB

Goal: AM = MD

#10



Given: AE = EB

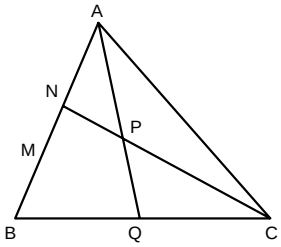
BF = FC

CG = GD

GF // HE

Goal: AH = HD

#11



Given: BQ = QC

AN = NM

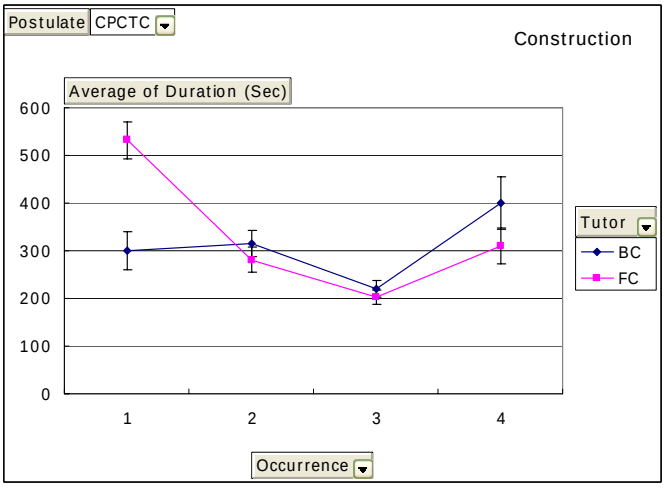
NM = MB

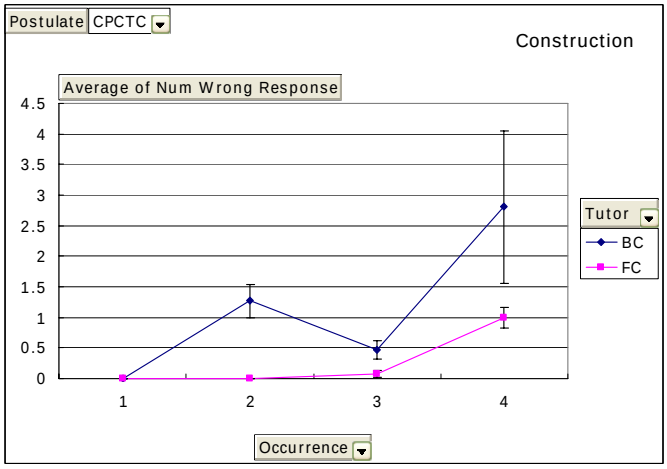
Goal: AP = PQ

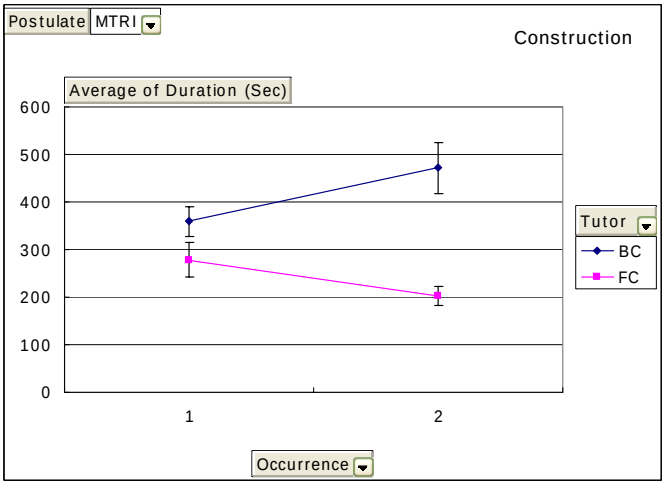
APPENDIX J: Learning Curves

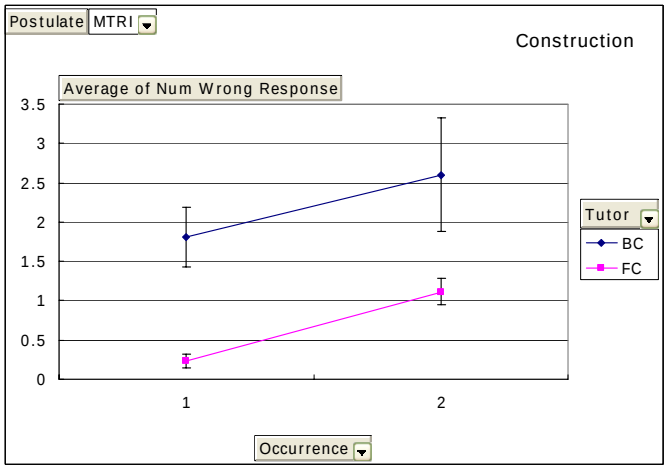
This appendix shows two types of learning curves: (1) average duration for postulate applications with and without construction, and (2) average number of error made during single postulate application with and without construction.

Postulate application with construction

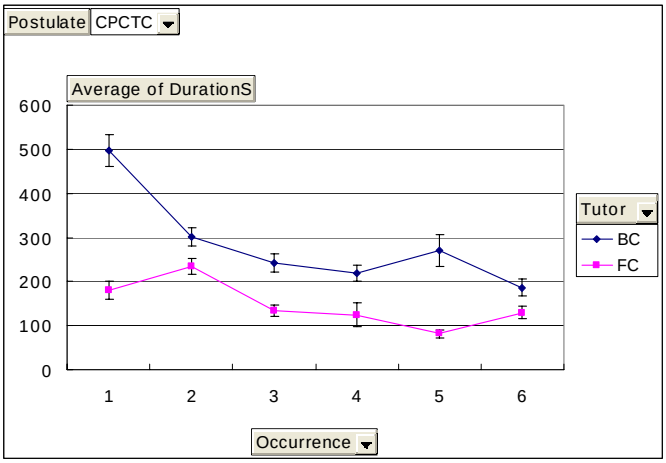


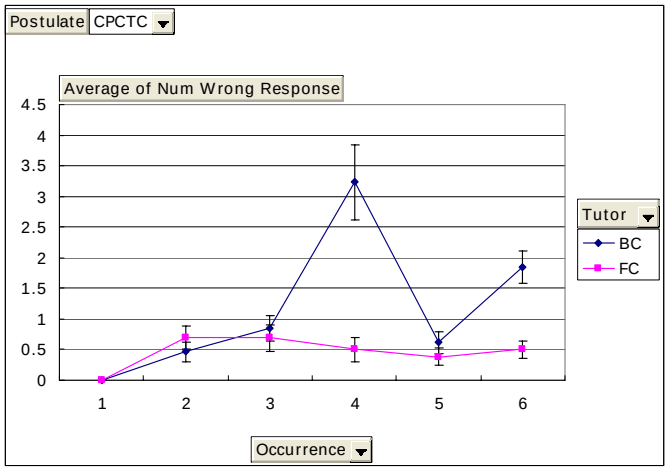


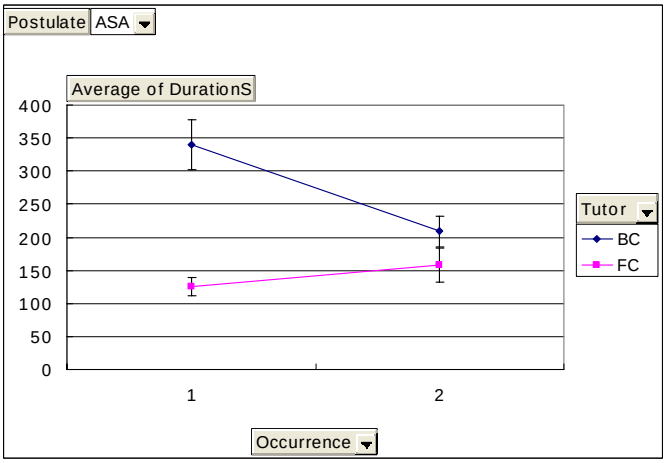


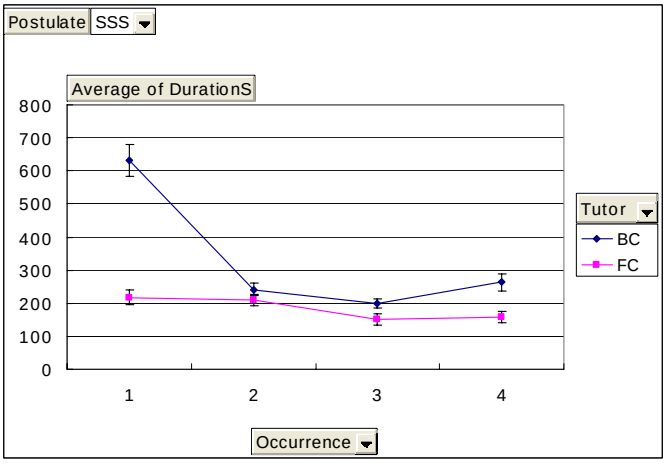


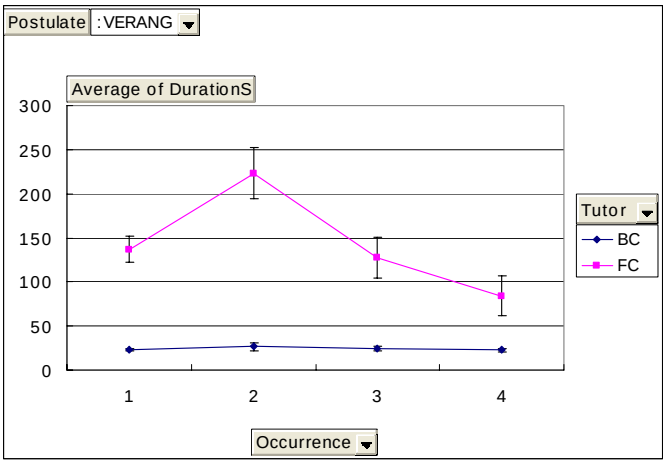
Postulate applications without construction

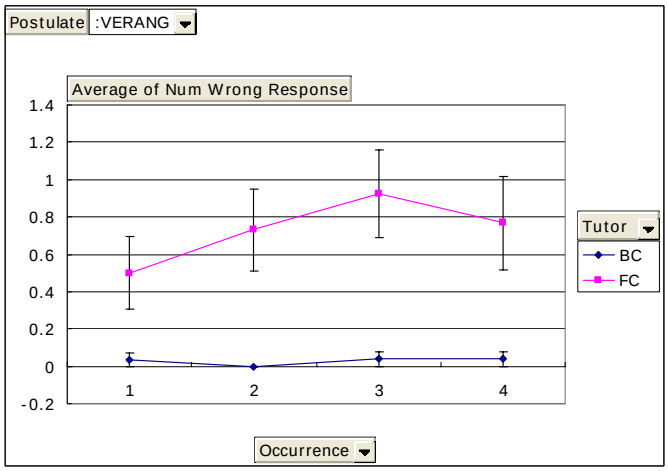


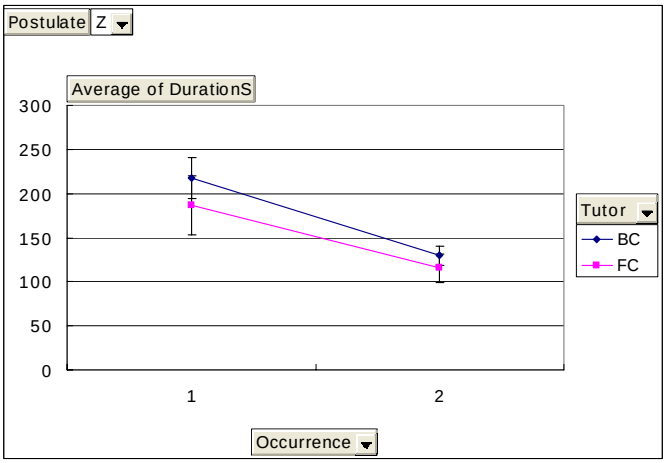







Postulate :VERANG
Average of DurationS
300
250
200
150
100
50
0
1
2
3
4
Occurrence
Tutor
BC
FC


Postulate :VERANG
Average of Num Wrong Response
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
1
2
3
4
Occurrence
Tutor
BC
FC


Postulate Z
Average of DurationS
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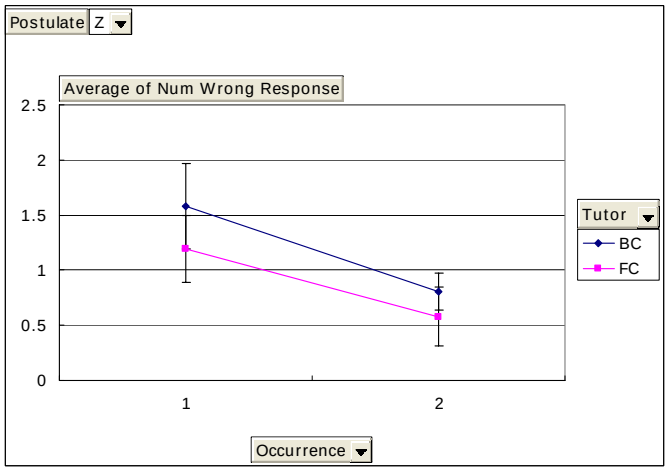
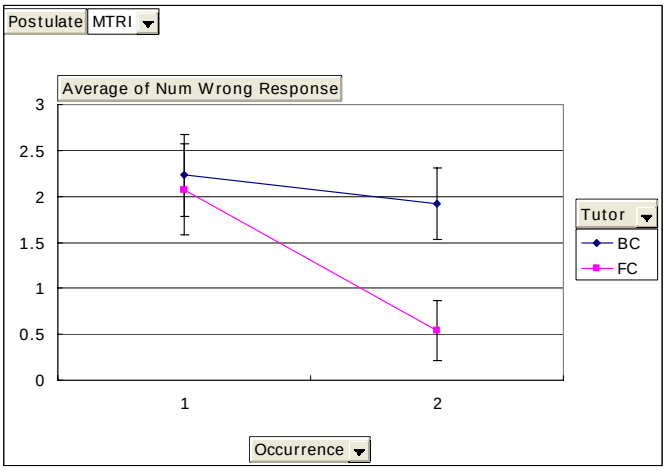
250
200
150
100
50
0
1
2
Occurrence
Tutor
BC
FC


Postulate Z
Average of Num Wrong Response
2.5
2
1.5
1
0.5
0
1
2
Occurrence
Tutor
BC
FC


Postulate MTRI
Average of Num Wrong Response
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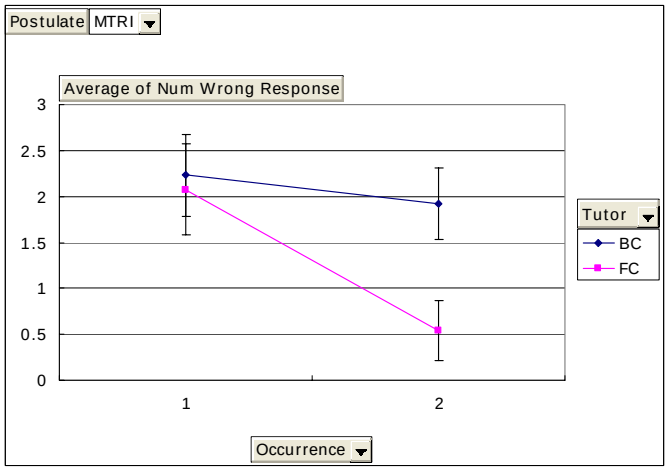
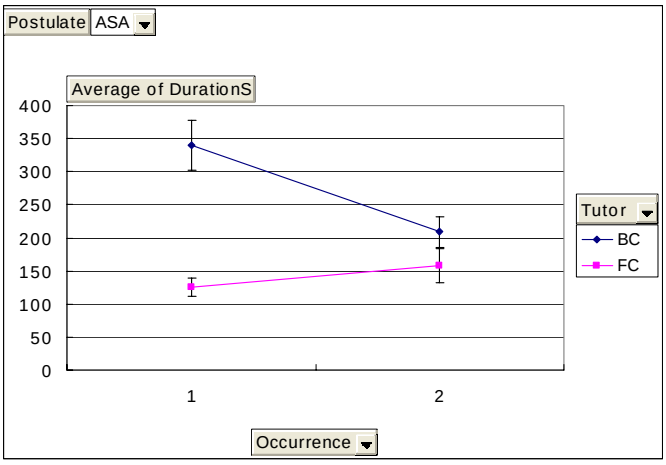
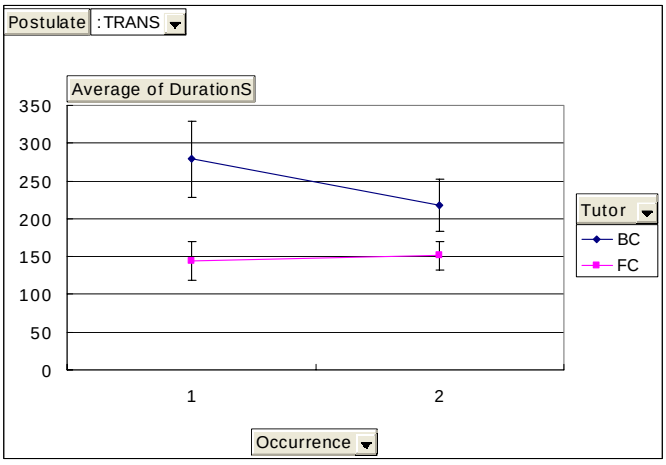
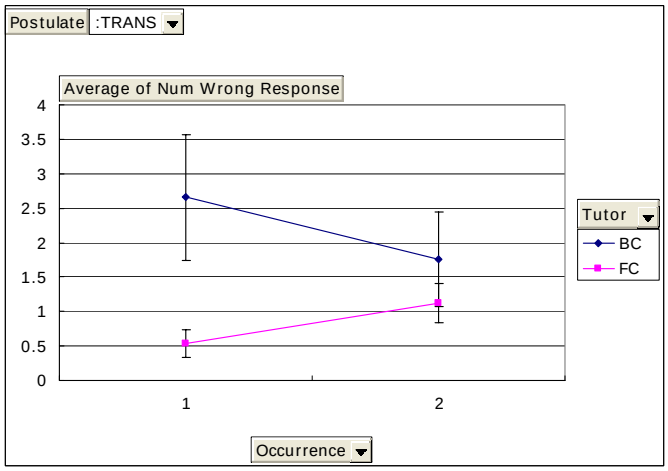
2.5
2
1.5
1
0.5
0
1
2
Occurrence
Tutor
BC
FC


Postulate MTRI
Average of Num Wrong Response
3
2.5
2
1.5
1
0.5
0
1
2
Occurrence
Tutor
BC
FC


Postulate ASA
Average of DurationS
400
350
300
250
200
150
100
50
0
1
2
Occurrence
Tutor
BC
FC


Postulate ASA
Average of Num Wrong Response
3.5
3
2.5
2
1.5
1
0.5
0
1
2
Occurrence
Tutor
BC
FC


Postulate :TRANS
Average of DurationS
350
300
250
200
150
100
50
0
1
2
Occurrence
Tutor
BC
FC


Postulate :TRANS
Average of Num Wrong Response
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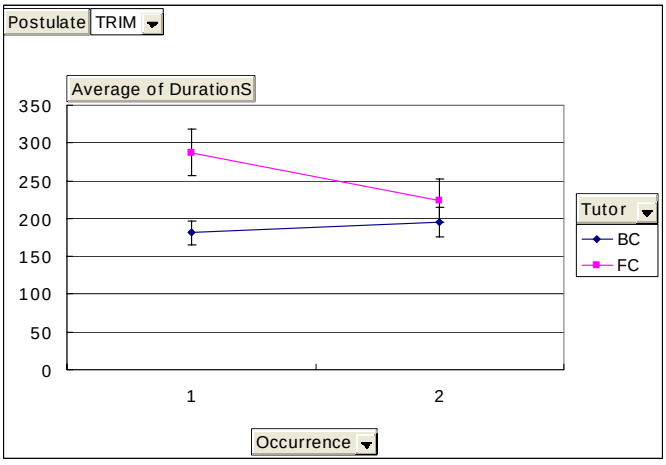
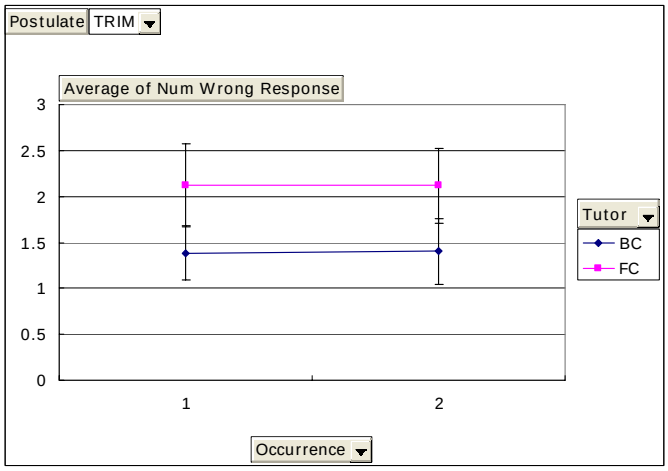
3.5
3
2.5
2
1.5
1
0.5
0
1
2
Occurrence
Tutor
BC
FC

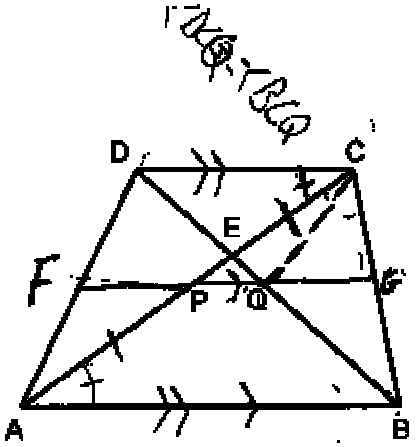

Postulate TRIM
Average of DurationS
350
300
250
200
150
100
50
0
1
2
Occurrence
Tutor
BC
FC


Postulate TRIM
Average of Num Wrong Response
3
2.5
2
1.5
1
0.5
0
1
2
Occurrence
Tutor
BC
FC

APPENDIX K: Example of Proofs written in an Inconsistent Strategy

6. Write a proof for the goal

Given: PQ // AB
CD // AB
AP = PC
Goal: DQ = QB



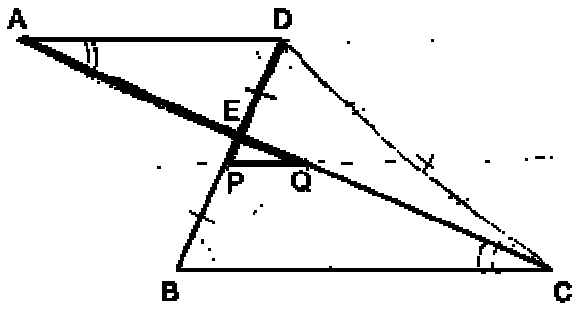
Proof:

	Proposition	Justification
1.	DQ = QB	
2)	PQ // AB	2) Given
3)	CD // AB	3) Given
4)	AP = PC	4) Given
5)	∠DEC = ∠AB	5) Z
6)	FG // DC	6) CoLPARA

Example 1: Beginning a proof by placing given propositions from the 2nd line.

6. Write a proof for the goal

Given: DP = PB
AD // BC
PQ // BC
Goal: AQ = QC



Proof:

	Proposition	Justification
1.	AQ = QC	
	AP // PQ	Transitive
	DP = PB	Given
	AD // BC	Given
	PQ // BC	Given

Example 2: Beginning a proof by placing given propositions at the bottom of the proof table.

APPENDIX L: Cognitive Model of Proof Writing

(a)

Forward chaining without construction

Assert Proposition

Justify Proposition

Select a postulate

Pick a postulate

Overlap configurations

Transform the postulate into a conditional form

Execute the postulate

Instantiate premises

Assert Line Numbers

(b)

Forward chaining with construction

Pick a postulate

Do construction

(c)

Backward chaining without construction

Select a proposition to justify

Apply a postulate backwards

Select a postulate

Pick a postulate

Overlap configurations

Transform the postulate into a conditional form

Execute the postulate

Instantiate premises

Check Duplication

Assert premises as unjustified propositions

Assert line numbers of the premises

(d)

Backward chaining with construction

Select a proposition to justify

Apply a postulate backwards

Select a postulate

Pick a postulate

Overlap configurations

Transform the postulate into a conditional form

Construction

Find missing segments

Construct missing segments

Execute the postulate

Instantiate premises

Check Duplication

Assert premises as unjustified propositions

Assert line numbers of the premises